

NOLTR 62-149

A COMPLETE E, P, V, T, S THERMODYNAMIC
DESCRIPTION OF METALS BASED ON THE
P, u MIRROR-IMAGE CONCEPT

NOI

13 AUGUST 1962

UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

20060608043

NOLTR 62-149

A Complete E,P,V,T,S Thermodynamic Description of
Metals Based on the P,u Mirror-Image Concept

Prepared by:

Julius W. Enig

ABSTRACT: The well-known practice of using the mirror-image, in the P,u plane, of the shock Hugoniot curve about a vertical line through the shocked state P_H, u_H is shown to give a complete thermodynamic description of metals when the U(shock velocity) vs. u_H relationship and $\alpha \equiv V^{-1}(\partial V/\partial T)_{P \approx 0}$ are known. Use of the experimental relationship $U = c_0 + au_H$ (a and c_0 constants) and $\alpha = \text{constant}$, leads to a thermodynamic description which results in the metal appearing less compressible than if described by the Mie-Grüneisen equation of state. Furthermore, the existence of an anomalous behavior of c_0 in the low pressure neighborhood (<10 to <50 kbar depending on the metal) of the initial state rules out the simultaneous existence of a Hugoniot satisfying the linear U vs. u_H relationship, of isentropes satisfying the mirror-image assumption, and of a constant value of α in this neighborhood. Thermodynamic functions for 16 metals are calculated up to 2 megabars and compared with the results obtained from the Mie-Grüneisen equation of state.

PUBLISHED SEPTEMBER 1962

APPROVED BY:

Donna Price, Acting Chief
Physical Chemistry Division
CHEMISTRY RESEARCH DEPARTMENT
U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, SILVER SPRING, MARYLAND

13 August 1962

This work was carried out under FR-59, Transition from Deflagration to Detonation. An important problem in this field is the lack of an adequate equation of state for solid materials. The work of this report is an original contribution in this area.

R. E. ODENING
Captain, USN
Commander

Albert J. Lightbody
ALBERT LIGHTBODY
By direction

TABLE OF CONTENTS

	<u>Page</u>
INTRODUCTION	1
THE HUGONIOT CURVE	6
THE E, P, V EQUATION OF STATE	7
CALCULATION OF T, S, AND c_p	11
RESULTS FOR 16 METALS	18
ACKNOWLEDGEMENTS	20
REFERENCES	21

TABLES

TABLE I	Input Data and Comparison with Zero-Pressure Data	22
TABLE II	Summary of Calculations and Results	23

FIGURES

FIGURE 1	Pressure versus particle velocity curves for an initial shock in a metal standard followed by a reflected shock or rarefaction wave, and a typical graphical solution to determine P_1, u_1 for a test specimen	25
FIGURE 2	Pressure versus volume curves for a metal that satisfies the mirror-image assumption	26

I. INTRODUCTION

A problem which is encountered quite often in experimental shock dynamics is the determination of the pressure and particle velocity behind the transmitted shock in a specimen after the initial shock, having traveled through a metal standard, has interacted with the interface between the specimen and the standard. An important application of this problem is the determination of the experimental shock Hugoniot of the specimen as a first step toward finding its equation of state. Consider a shock traveling to the right through a metal standard whose initial conditions (given by the subscript o) are $u_o = 0$, E_o , V_o , $P_o \approx 0$, where u, E, V, P are respectively, the particle velocity, specific internal energy, specific volume, and pressure. A pressure P_o of at most a few bars is considered negligible in present considerations since the pressures of interest are in the kilobar to megabar range. When this shock interacts with the interface a shock is transmitted into the specimen and a wave traveling to the left is reflected into the standard. The loci of P, u points that can be reached by the initial shock and the reflected wave in the standard are shown in Fig. 1. For pressures greater and particle velocities smaller than that corresponding

to the initial shock, the reflected wave is a reflected shock. For smaller pressures and greater particle velocities, the reflected wave is a reflected rarefaction wave. The shock equation

$$P_H = \rho_0 U u_H \quad (1)$$

relates the shock pressure P_H , shock velocity U , shock particle velocity u_H , and initial density ρ_0 ($=1/V_0$). From now on the subscript H will always refer to values on the shock Hugoniot. Hence the measurement of ρ_0 and U (of the transmitted shock) in the specimen implies that P_H and u_H (in the specimen) lie on the straight line of known slope $\rho_0 U$ as given by Eq. (1). If this is depicted by the dashed line in Fig. 1, then the intersection with the rarefaction wave in the standard is the desired pressure P_1 and particle velocity u_1 behind the transmitted shock in the specimen (since P and u are continuous at the interface).

It will always be assumed that the effects of material rigidity can be neglected since the shock pressures are far greater than the yield point of the metals involved. Hence the metal can be treated as a fluid with an ordinary equation of state.

The calculation of the rarefaction waves (isentropes) entails a knowledge of the E, P, V equation of state.

Walsh and Christian¹ computed the temperatures along the Hugoniot and the isentropes for Al, Cu and Zn from the Hugoniot curves combined with the assumption that the specific heat at constant volume c_v and $(\partial P/\partial T)_V$ are constants. Walsh and Rice² noted that at least for Al the changes in the P,u isentropes are insensitive to changes in the value of $(\partial P/\partial E)_V$. Walsh, et al.,³ employed the Mie-Grüneisen equation of state for which the volume dependence of the Grüneisen ratio was determined using the Dugdale-MacDonald relation. These considerations allowed them to compute the isentropes in the neighborhood of the experimental Hugoniot curve. Additional zero-pressure data then permitted the calculation of the remaining thermodynamic data of interest, and numerical results were tabulated for 27 metals. McQueen and Marsh⁴ applied the previous theory to more extensive experimental data in the one-to-two megabar region for 16 metals. Their experimental shock measurements actually covered 19 metallic elements. Al'tshuler, et al.,^{5,6} have reported investigations of iron to shock pressures of about 5 megabars, and 3 to 5 megabars for Ag, Au, Bi, Cd, Cu, Pb, Sn, and Zn. The Mie-Grüneisen equation of state, in which the Grüneisen ratio was considered to be a constant, was used to describe iron. Further work⁷ on Al, Cu, and Pb took into account the

electronic components of the energy and pressure.

Consider for a moment that the specimen mentioned in the problem above is replaced by a vacuum (approximating air at several bars pressure). When the plane shock in the metal which connects the state $P_0 \approx 0$, $u_0 = 0$ and P_H, u_H arrives at a free surface, the pressure in the shock is reduced to zero by a rarefaction wave from the surface. Let u_{fs} be the free surface particle velocity. It was first postulated by Goranson⁸ about 1945 that for most metals

$$2 u_H \approx u_{fs}. \quad (2)$$

Thus experimental values of U and u_{fs} determine the Hugoniot. In fact this approximation is the first guess in a rapidly convergent iterative procedure for bounding¹ and determining^{2,3,4} the Hugoniot when additional thermodynamic information is given. Tabulated results^{1,3,4} show that this approximation is fairly accurate for many metals.

For metals a further assumption (which includes Eq. (2) as a special case) is frequently made. The isentropic expansion from P_H, u_H to $0, 2u_H$ is approximated by taking the mirror-image of the Hugoniot curve about a vertical line through P_H, u_H in the P, u plane (see, for example, Refs. 9 and 10). This makes it particularly simple to calculate P_i, u_i . One need have only the Hugoniot

P, u curve for the metal, reflect it about the appropriate point corresponding to the measured shock velocity in the metal, and find the intersection of this reflected curve with the straight line given by Eq. (1) (dashed line of Fig. 1). Under this assumption the curve of an isentropic compression from the point $u_0 = 0, E_0, P_0 \approx 0, V_0$ coincides with the Hugoniot in the P, u plane, but intersects it only once in the P, V plane. Hence the Hugoniot is not an isentrope as one might be apt to think at first glance.

Since the mirror-image assumption is often used it is worth examining its consequences, and in the remainder of this paper it will be shown that this assumption and the experimental U, u_H relation are enough to determine a unique E, P, V equation of state in the metal. Furthermore an experimental thermal coefficient of volume expansion along the isobar $P_0 \approx 0$ determines the temperature T , entropy S , and other thermodynamic properties as functions of any two thermodynamic variables, e.g., P and V . In Section II the Hugoniot relations valid for many metals are given. An exact though complicated E, P, V equation of state which is a direct consequence of the mirror-image assumption is derived in Section III. Section IV contains the derivation of the temperature, entropy, and specific heat at constant

pressure. The anomalous behavior of the latter in the low pressure region where the Hugoniot is not well defined is discussed. Finally, in Section V, tabulated results for various thermodynamic quantities for 16 metals based on the mirror-image approximation are compared with results obtained by McQueen and Marsh⁴ who utilized the more sophisticated Mie-Grüneisen equation of state.

II. THE HUGONIOT CURVE

Many investigators have found that for many metals the shock velocity is linear in the shock particle velocity, i.e.,

$$U = c_o + a u_H. \quad (3)$$

Theoretically, c_o should be the adiabatic sound speed at the initial state $E_o, P_o, V_o, u_H = 0$. Least-square straight-line fits of the experimental U, u_H data⁴ give as the intercepts c_o values that are quite close to the isentropic sound speeds at P_o, V_o ,

$$c_o^{(B)} = V_o \left(-\frac{\partial P}{\partial V} \right)_S^{\frac{1}{2}} = \lim_{u_H \rightarrow 0} U = \lim_{P_H \rightarrow P_o} V_o \left(\frac{P_H - P_o}{V_o - V_H} \right)^{\frac{1}{2}}, \quad (4)$$

which were computed⁴ from Bridgeman's data by applying a small correction for the difference between isothermal and isentropic first derivatives.

Elimination of U between Eqs. (1) and (3) yields

$$P_H = \rho_o u_H (au_H + c_o). \quad (5)$$

From the other two relations valid on the Hugoniot,

$$u_H^2 = P_H (v_o - v_H), \quad (6)$$

$$E_H - E_o = \frac{1}{2} P_H (v_o - v_H), \quad (7)$$

v_H and E_H can be written as functions of u_H :

$$v_o - v_H = v_o u_H / (au_H + c_o), \quad (8)$$

$$E_H - E_o = \frac{1}{2} u_H^2. \quad (9)$$

From Eqs. (6) and (8), the equation of the Hugoniot in the P, V plane is

$$P_H = c_o^2 (v_o - v_H) / [(a-1)v_o - av_H]^2; \quad (10)$$

the slope of the Hugoniot is

$$dP_H / dv_H = -\rho_o^2 (2au_H + c_o)(au_H + c_o)^2 / c_o < 0. \quad (11)$$

If Eq. (3) holds for all shock strengths, then letting $P_H \rightarrow \infty$ in Eq. (10) leads to

$$\rho_{max}/\rho_o = v_o/v_{min} = a/(a-1), \quad a > 1, \quad (12)$$

where ρ_{max} ($= 1/v_{min}$) is the maximum density that can be obtained with a single shock starting at ρ_o .

III. THE E,P,V EQUATION OF STATE

If $P_H = h(u_H)$ refers to the Hugoniot pressure-particle velocity curve, and $P(u; u_H)$ refers to the pressure-particle velocity curve for an isentropic expansion from a point P_H, u_H , then the assumption that the expansion from the shocked state P_H, u_H is the mirror

image of the Hugoniot about a vertical line through P_H, u_H is expressed by

$$P(u; u_H) = h(2u_H - u), \quad (13)$$

where u_H is now a continuous parameter but constant for each isentrope. Actually u_H is a single-valued function of the entropy; the explicit relationship is derived in Section IV. By Eq. (5),

$$P(u; u_H) = \rho_0(2u_H - u) [a(2u_H - u) + c_0]. \quad (14)$$

The solution for u as a function of P is

$$2u_H + \frac{c_0}{2a} - u = \left[\frac{P}{\rho_0 a} + \left(\frac{c_0}{2a} \right)^2 \right]^{\frac{1}{2}}, \quad (15)$$

where the positive root is taken for $u \leq 2u_H$. Since

$$dP(u; u_H)/du = \rho_0(2au - 4au_H - c_0), \quad (16)$$

the equation,

$$(du)^2 + dPdV = 0, \quad (17)$$

valid on an isentrope, can now be integrated between the points u_H, V_H and u, V to give

$$2\rho_0 a(V_H - V) = \ln \left[1 + 2a(u_H - u)/(2au_H + c_0) \right]. \quad (18)$$

Elimination of u between Eqs. (15) and (18) leads to the isentropic equation

$$4aP(V; u_H) = (4aP_H + \rho_0 c_0^2) e^{-4a\rho_0(V - V_H)} - \rho_0 c_0^2. \quad (19)$$

Define a new parameter

$$z = au_H/c_0 = (U - c_0)/c_0. \quad (20)$$

With the use of Eqs. (5), (8), and (20), Eq. (19) can be rewritten as

$$\begin{aligned} \Psi(P, V) &\equiv \left(1 + 4aV_0 P/c_0^2\right)^2 e^{-4a\rho_0(V-V_0)} = \\ &= (2z+1)^2 e^{-4z/(z+1)} \equiv w(z), \end{aligned} \quad (21)$$

where $\Psi(P, V)$ and $w(z)$ are defined as, respectively, the left-and right-hand sides of Eq. (21). Using Eq. (19), the adiabatic sound speed c can be obtained from

$$\rho^2 c^2 \equiv -(\partial P / \partial V)_S = 4a\rho_0 P + \rho_0^2 c_0^2, \quad (22)$$

which defines the slopes of the P, V isentropes¹¹

The energy along the isentrope is found from

$$E(V; u_H) - E_H = - \int_{V_H}^V P(V'; u_H) dV' \quad (23)$$

to be,

$$4a [E(V; u_H) - E_H] - \rho_0 c_0^2 (V - V_H) - V_0 [P(V; u_H) - P_H] = 0 \quad (24)$$

where Eq. (19) has been used. In more convenient form, Eq. (24) can be written as

$$\begin{aligned} \left(\frac{c_0}{2a}\right)^2 \Phi(E, P, V) &\equiv E - E_0 - \frac{\rho_0 c_0^2}{4a} (V - V_0) - \frac{PV_0}{4a} = \\ &= E_H - E_0 - \frac{\rho_0 c_0^2}{4a} (V_H - V_0) - \frac{P_H V_0}{4a}. \end{aligned} \quad (25)$$

Since the right-hand side of Eq. (25) is just a function of u_H ,

$$\bar{\Phi}(E, P, V) = z^3/(z+1) \geq 0. \quad (26)$$

If z can be eliminated between Eqs. (21) and (26), the resulting equation would define the desired E, P, V equation of state. This may be accomplished by solving Eq. (26),

$$z^3 - \bar{\Phi}z - \bar{\Phi} = 0,$$

for the single positive root $z(\bar{\Phi})$ and then substituting this into Eq. (21). In order to find the root the following two cases must be considered:

(i) If $4\bar{\Phi} \leq 27$, then the positive root is given by

$$z = \left(\frac{1}{2}\bar{\Phi}\right)^{\frac{1}{3}} \left\{ \left[1 + \left(1 - \frac{4}{27}\bar{\Phi}\right)^{\frac{1}{2}} \right]^{\frac{1}{3}} + \left[1 - \left(1 - \frac{4}{27}\bar{\Phi}\right)^{\frac{1}{2}} \right]^{\frac{1}{3}} \right\}. \quad (27)$$

(ii) If $4\bar{\Phi} \geq 27$, then the positive root is given by

$$z = 2\left(\bar{\Phi}/3\right)^{\frac{1}{2}} \cos\left(\frac{1}{3}\theta\right), \quad (28)$$

where

$$0 < \cos\theta = \frac{1}{2}\left(27/\bar{\Phi}\right)^{\frac{1}{2}} \leq 1, \quad 0 \leq \theta < \frac{\pi}{2}. \quad (29)$$

Hence the isentrope characterized by $z = 3$ (i.e., the isentrope that intersects the Hugoniot when $U = 4 c_0$) divides the P, V plane into two regions. Eq. (27) holds above this isentrope, Eq. (28) below. Therefore Eqs. (21), (27), (28), and (29) define the E, P, V equation of state, i.e.,

$$\Psi(P, V) = w(z[\bar{\Phi}(E, P, V)]). \quad (30)$$

IV. CALCULATION OF T, S, AND c_p

The derivation of the thermal equation of state which gives the temperature as a function of pressure and volume proceeds in the following manner. Along an isentrope,

$$TdS = c_v dT + T(\partial P/\partial T)_V dV = 0, \quad (31)$$

where c_v is the specific heat at constant volume. Since

$$c_v (\partial P/\partial E)_V = (\partial P/\partial T)_V, \quad (32)$$

the integral of Eq. (31) is obtained from

$$\int_{T(0,V)}^{T(P,z)} \frac{dT'}{T'} = - \int_0^P \left(\frac{\partial P'}{\partial E} \right)_V \left(\frac{\partial V}{\partial P'} \right)_S dP'. \quad (33)$$

The lower limit of the first integral, $T(0,V)$, is found from the experimental coefficient of volume expansion along the isobar $P \approx 0$. Let $T(0,V) = T_0 f(V/V_0)$, where T_0 is the initial temperature corresponding to E_0, V_0, P_0 , and f is an arbitrary function of V/V_0 . From Eq. (21),

$$g(z) \equiv \frac{V}{V_0} = 1 + \frac{1}{2a} \ln (2z+1) - \frac{z}{a(z+1)} \text{ on } P \approx 0. \quad (34)$$

Define $F(z) \equiv T(0,V)/T_0 = f[g(z)]$. Differentiation of Eq. (30) with respect to P gives

$$(\partial E/\partial P)_V = \left[(\partial \bar{\Phi}/\partial z)(dz/dw)\partial \bar{\Psi}/\partial P - \partial \bar{\Phi}/\partial P \right] \partial E/\partial \bar{\Phi}, \quad (35)$$

and evaluation of the derivatives leads to

$$\left(\frac{\partial E}{\partial P} \right)_V = \frac{V_0}{4a} \left[1 + \frac{(2z+3)(2z+1)}{1+4aPV_0/c_0^2} \right]. \quad (36)$$

Substitution of Eqs. (22) and (36) into Eq. (33) gives the result,

$$T(P;z)/T_0 = F(z) \left\{ 1 + aP / \left[\rho_0 c_0^2 (z+1)^2 \right] \right\}, \quad (37)$$

on the isentrope characterized by the value of z . Eqs. (21) and (37) determine the P, V, T equation of state through the parameter z .

By Eqs. (25) and (34),

$$4a^2 [E(z) - E_0]^2 / c_0^2 = z(z-1) + \frac{1}{2} \ln(2z+1) \text{ on } P \approx 0. \quad (38)$$

On the isobar $P \approx 0$, the entropy S can be calculated from

$$T_0(S-S_0) = \int_0^z \frac{1}{F(Z)} \frac{dE(Z)}{dZ} dZ. \quad (39)$$

Hence the evaluation of Eq. (39) relates S to z , namely,

$$\frac{a^2 T_0}{c_0^2} [S(z) - S_0] = \int_0^z \frac{z^2 dz}{(2z+1)F(Z)}. \quad (40)$$

The positiveness of the integrand implies that S is a single-valued function of z . Of course this relation holds not only on $P \approx 0$ but on all paths connecting the isentropes S_0 and S .

A complete E, P, V, T, S thermodynamic description of the metal has now been obtained. For example, given the point P_1, V_1 , the parameter z_1 can be obtained from Eq. (21) by simple numerical methods. Then E_1, T_1 , and S_1 are computed respectively from Eqs. (25), (26), (37), and (40); in

general S_1 necessitates a numerical integration. The equations of state derived above hold in the region of the P,V plane that lies to the right of the isentrope through the point P_o, V_o, E_o in the direction of increasing entropy. If a Hugoniot of only finite length is considered (because the experimental data is limited), then the region of definition of the above equations of state is limited to the region bounded by the isentropes that pass through the lower and upper points of the Hugoniot (see Fig. 2).

The specific heat at constant pressure,

$$c_p \equiv T(\partial S/\partial T)_P = T(dS/dz)(\partial z/\partial T)_P, \quad (41)$$

found by differentiation of Eqs. (37) and (40), is

$$\frac{a^2 T_o}{c_o^2} c_p(P, z) = \frac{z^2(z+1)[aP + p_o c_o^2 (z+1)^2]}{(2z+1)\{(z+1)[aP + p_o c_o^2 (z+1)^2]dF/dz - 2aPF(z)\}} \quad (42)$$

In particular on the isobar $P \approx 0$,

$$(a/c_o)^2 T_o c_p(0, z) = z^2 / [(2z+1)dF/dz]. \quad (43)$$

Thus far the temperature variation with volume for $P \approx 0$ has been given as an arbitrary function of the entropy, $F[z(S)]$. In order to find the explicit form the thermal coefficient of volume expansion α will be considered as constant on $P \approx 0$, i.e.,

$$V^{-1}(\partial V/\partial T)_{P \approx 0} \equiv \alpha = \text{constant} \quad (44)$$

where the constant is experimentally determined.

Integration of Eq. (44) and use of Eq. (34) leads to the result,

$$F(z) = 1 + \frac{1}{\alpha T_0} \ln \left[1 + \frac{1}{2a} \ln(2z+1) - \frac{z}{a(z+1)} \right], \quad (45)$$

from which it follows that

$$\frac{a}{\alpha c_0^2} c_p(P, z) = \frac{(z+1)^2 g(z)}{1-2a^2 \alpha T_0 (z+1)(2z+1) F(z) P / [az^2 P + p_0 c_0^2 z^2 (z+1)^2]}, \quad (46)$$

$$c_p(0, z) = \alpha c_0^2 (z+1)^2 g(z) / a. \quad (47)$$

Since, with the use of Eq. (21),

$$\alpha T_0 F(z) = \alpha T_0 - \ln(4a) + \ln[4a + \ln \Psi(P, V)], \quad (48)$$

Eq. (37) can be solved for z with the aid of Eq. (48), yielding

$$z = \left(\frac{aP}{p_0 c_0^2} \right)^{\frac{1}{2}} \left[\frac{\alpha T}{\alpha T_0 - \ln(4a) + \ln[4a - \ln \Psi(P, V)]} \right]^{-\frac{1}{2}} - 1. \quad (49)$$

Substitution of Eq. (49) into Eq. (21) leads to an exact though quite complicated P, V, T equation of state.

Close examination of Eq. (46) shows that c_p exhibits certain very undesirable behavior. First $c_p(P, 0) = 0$ for $P > 0$, i.e., c_p vanishes on the isentrope passing through the point E_0, P_0, V_0 . Furthermore, the denominator of Eq. (46) vanishes on the curve C (dashed curve in Fig. 2) defined by

$$P(z) = p_0 c_0^2 z^2 (z+1)^2 / [2a^2 \alpha T_0 (z+1)(2z+1) F(z) g(z) - az^2], \quad (50)$$

implying that $c_p \rightarrow \infty$ or $-\infty$ in the neighborhood of C depending on the direction of approach. This curve intersects the Hugoniot at the values of z given by the roots of

$$z = 2a\alpha T_0(z+1)F(z)g(z). \quad (51)$$

The lower point of intersection z_l can be approximated by

$$z_l \approx 2a\alpha T_0 \ll 1; \quad (52)$$

the upper point of intersection occurs at pressures which are much greater than the upper limit of the experimental shock pressures and therefore need not be considered.

The shock pressure at which $c_p \rightarrow \pm \infty$ is

$$P_H(z_l) \approx 2\rho_0 c_0^2 \alpha T_0, \quad (53)$$

which for the metals considered is in the range of 10 to 50 kilobars (see Table I). Certainly the equations of state derived above are not valid in the neighborhood of C (i.e., where $c_p \rightarrow \pm \infty$) or in the region (see Fig. 2) to the left of C (i.e., where $c_p \leq 0$). Numerical computations for the 16 metals show that for pressures up to at least 5 megabars (and in most cases much higher) the pressure increases monotonically with increasing z and decreasing V on the curve C. Furthermore for $P < P_H(z_l)$ and $P > P_H(z_l)$, C lies respectively to the right and to the left of the Hugoniot (as shown in Fig. 2). Along the Hugoniot, the specific heat $c_{p,H}$ rapidly decreases from ∞ at $P_H(z_l)$ to a minimum (at which the pressure is at most 0.2 megabars as can be seen in Table II) and then slowly increases.

The anomalous behavior of c_p leads to the conclusion that it is impossible to postulate the simultaneous existance of (i) a Hugoniot satisfying Eq. (3) and (ii) of isentropes satisfying the mirror-image assumption, both in the low pressure neighborhood of P_0, V_0, E_0 , and (iii) of a constant thermal coefficient of volume expansion along $P \approx 0$. Walsh, et al, using the Mie-Grüneisen equation prescribed in addition to (i) and (iii), the experimental values for c_p along $P \approx 0$. Since the Gruneisen ratio was taken as a function of V only it was easy to show that c_p was a function of S only. Therefore c_p was well-behaved in their calculations. The conclusion is that the mirror-image assumption in the neighborhood of the low pressure Hugoniot given by Eq. (3) leads to the anomalous behavior of c_p . However it is just in this region that it is not possible to determine experimentally a unique Hugoniot. For sufficiently low pressures the effects of material rigidity (which have been neglected in the above analysis) give rise to elastic-plastic wave structures. Here an arbitrary assumed shock wave will break up into two compression waves, an elastic wave which precedes a plastic wave. This "two-wave" structure indicates that at some point on the Hugoniot locus of P, V states, there is a violation of the Bethe-Weyl necessary condition $(\partial^2 P / \partial V^2)_S > 0$, for the existance of a stable shock in a single phase fluid.

A. Special Results on the Hugoniot

It is readily seen that on the Hugoniot curve,

$$P_H = \rho_0 c_0^2 z(z+1)/a, \quad (54)$$

$$V_H/V_0 = 1 - z/[a(z+1)], \quad (55)$$

$$E_H = E_0 + \frac{1}{2} c_0^2 z^2/a^2, \quad (56)$$

$$T_H = T_0 F(z)[1+z/(z+1)], \quad (57)$$

$$U = c_0(z+1), \quad (58)$$

$$c_{p,H} = \frac{\alpha c_0^2 z(z+1)^3 g(z)/a}{z-2\alpha T_0(z+1)g(z)F(z)}, \quad (59)$$

$$c_H = c_0(2z+1)(1-z/[a(z+1)]), \quad (60)$$

V. RESULTS FOR 16 METALS

In Sections III and IV it was shown that under the mirror-image assumption a complete thermodynamic description of a metal depended only on the experimental values of c_o and a , and the coefficient of volume expansion along the isobar, $P \approx 0$. These data are listed in Table I. The U, u_H curves of the 16 metals of Table I were obtained⁴ by applying the method described at the very beginning of Section I. Different known shock velocities in a brass standard, whose equation of state is of the Mie-Grüneisen form, give rise to different measured shock velocities in the specimen (any one of the 16 metals), and hence to a U, u_H locus for the specimen. Columns 7 and 8 of Table I compare the specific heat $c_p = \alpha c_o^2/a$ at P_o, V_o (see Eq. (47)) obtained from the mirror-image assumption with the handbook values $c_p^{(3)}$ listed by Walsh, et al.³

A summary of calculations and results based on this assumption is given in Table II and these results may be compared with those obtained by McQueen and Marsh using the Mie-Grüneisen equation of state (see Table IV of Ref. 4). It is found that the mirror-image assumption always leads to smaller values of the free-surface volume (i.e., at $P \approx 0$) resulting from expansion from the shocked state. For example, for copper, corresponding to the entries of column

10 of Table II, the Mie-Grüneisen equation gives for the free-surface volumes, $V/V_0 = 1.000, 1.000, 1.001, 1.003, 1.006, 1.010, 1.015, 1.019, 1.025, 1.031, 1.037, 1.044, 1.051, 1.058, 1.066, \text{ and } 1.079$. Furthermore, along the isentropes tabulated, V/V_0 is always smaller for identical pressures except in the case of thallium (see columns 14 and 15 of Table II); and for identical P the differences between the compressions, $[V(P,z) - V_H(z)]/V_H(z)$, calculated with each equation of state (starting from the same $P_H(z)$, $V_H(z)$), are smallest at high pressures (particularly for Mo, Ni, Ti, V, and W) and greatest at low pressures. As a consequence the temperatures, both along the Hugoniot and along the isentropes, are also lower (and for some metals very much lower) with the mirror-image assumption¹². For example, for copper, corresponding to the entries of column 4 of Table II, the Mie-Grüneisen equation gives for the Hugoniot temperatures (in °K), $T_H = 293, 334, 391, 472, 582, 719, 881, 1068, 1277, 1506, 1755, 2020, 2301, 2596, 2902, \text{ and } 3042$.

The results of these computations demonstrate that the thermodynamic description obtained by the mirror-image assumption causes the metal to appear less compressible than when the thermodynamic description is obtained from the Mie-Grüneisen equation of state.

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge the many fruitful discussions with Dr. Donna Price and Dr. S. Jacobs of the U.S. Naval Ordnance Laboratory. The author is grateful to Dr. R. G. McQueen of the Los Alamos Scientific Laboratory for supplying the numerical data listed in column 14 of Table II.

REFERENCES

- (1) J. M. Walsh and R. H. Christian, Phys. Rev. 97, 1544 (1955).
- (2) J. M. Walsh and M. H. Rice, J. Chem. Phys. 26, 815 (1957).
- (3) J. M. Walsh, M. H. Rice, R. G. McQueen, and F. L. Yarger, Phys. Rev. 108, 196 (1957).
- (4) R. G. McQueen and S. P. Marsh, J. Appl. Phys. 31, 1253 (1960).
- (5) I. V. Al'tshuler, K. K. Krupnikov, B. N. Ledenev, V. I. Zhuchikhin, and M. I. Brazhnik, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 874 (1958); Soviet Phys. JETP 34(7), 606 (1958).
- (6) L. V. Al'tshuler, K. K. Krupnikov, and M. I. Brazhnik, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 886 (1958); Soviet Phys. JETP 34(7), 614 (1958).
- (7) L. V. Al'tshuler, S. B. Kormer, A. A. Bakanova, and R. F. Trunin, J. Exptl. Theoret. Phys. (U.S.S.R.) 38, 790 (1960); Soviet Phys. JETP 11, 573 (1960).
- (8) R. W. Goranson, D. Bancroft, B. L. Burton, T. Blechar, E. E. Houston, E. F. Gittings, and S. A. Landeen, J. Appl. Phys. 26, 1472 (1955).
- (9) A. W. Campbell, W. C. Davis, J. B. Ramsay, and J. R. Travis, Phys. Fluids 4, 511 (1961).
- (10) G. E. Seay and L. B. Seeley, Jr., J. Appl. Phys. 32, 1092 (1961).
- (11) S. Jacobs pointed out that the isentropes in the P, V plane can be generated from any one isentrope, e.g., the one passing through P_0, V_0, E_0 by translating it in the V -direction. This can readily be seen by noting that Eq. (22) is a constant on any isobar. The isentropes in the P, u plane can also be generated by a translation in the u -direction.
- (12) While in this work the coefficient of volume expansion α is taken as a constant for each metal, it was not in Ref. 4. This has some effect on the computed temperatures but accounts for only a small part of the differences between the temperatures obtained from the mirror-image and Mie-Grüneisen equations of state.

Table I. Input data and comparison with zero-pressure data. The values of ρ_0 , c_0 , a , and $c_0^{(B)}$ are taken from Reference 4 (see their Table III); and the thermal coefficient of volume expansion α and the specific heat $c_p^{(3)}$ are handbook values from Reference 3 (see their Table I), except as indicated. The quantity $c_p(0,0) = \alpha c_0^2/a$ appearing in column 8 is the theoretical specific heat obtained by letting $z=0$ in Eq. (47). The quantities $c_0^{(B)}$, $c_p^{(3)}$, and $c_p(0,0)$ are values at the initial state $P_0 \approx 0$, V_0 . $P_H(z_l)$, the calculated shock pressure at which $c_p \rightarrow 0$, is computed from Eqs. (51) and (54).

Material	ρ_0 g/cm ³	$\alpha \times 10^6$ $/^\circ\text{K}$	a	c_0 cm/ μsec	$c_0^{(B)}$ cm/ μsec	$c_p^{(3)}$ (cal/g) $/^\circ\text{K}$	$c_p(0,0)$ (cal/g) $^\circ\text{K}$	$P_H(z_l)$ kiloba.
Ag	10.49	56.7	1.586	0.3243	0.319	0.056	0.090	40.9
Au	19.24	42.6	1.560	0.3075	0.305	0.031	0.043	49.2
Cd	8.64	89.4	1.671	0.2443	0.241	0.055	0.076	32.6
Co	8.82	36.9	1.330	0.4748	0.463	0.099	0.149	45.6
Cr	7.10	18.6	1.465	0.5217	0.515	0.111 ^a	0.083	21.7
Cu	8.90	49.5	1.497	0.3958	0.398	0.092	0.124	44.2
Mo	10.20	15	1.238	0.5157	0.519	0.061	0.077	24.4
Ni	8.86	39	1.445	0.4646	0.463	0.105	0.139	46.8
Pb	11.34	85.1	1.517	0.2028	0.202	0.030	0.055	27.3
Sn	7.28	60	1.476	0.2640	0.276	0.054	0.068	19.9
Th	11.68	33.3	1.278	0.2132	0.205	0.030	0.028	10.9
Tl	4.51	25.5	1.089	0.4779	0.484	0.126	0.128	15.9
Tl	11.84	114	1.515	0.1859	0.183	0.031	0.062	34.1
V	6.1	23.4 ^b	1.210	0.5108	0.518	0.115 ^a	0.121	22.6
W	19.17	17.4 ^b	1.268	0.4005	0.405	0.032 ^a	0.053	32.2
Zn	7.14	100	1.559	0.3050	0.303	0.092	0.143	47.5

^a Smithsonian Physical Tables, prepared by W. E. Forsythe (Smithsonian Institution, Washington, D.C., Ninth Revised Edition, 1954), pp. 155, 157

^b Reference a, p. 146

TABLE II. Summary of calculations and results. The columns headed "shock wave parameters" refer to hydrodynamic and thermodynamic quantities associated with the various shock pressures listed in column 2. The columns headed "isentropic release-wave parameters at $P \approx 0$ " refer to the hydrodynamic and thermodynamic quantities of the material relieved isentropically to zero pressure from P_H . The last three columns are the relative volume^a computed from the Mie-Gruneisen equation of state ($\gamma(G)/V_0$), and the relative volume and temperature from the mirror-image assumption, for the isentrope that intersects the Hugoniot at the pressure corresponding to the smallest relative volume in column 14 (and 15); the pressure P in the second column now refers to the corresponding pressure for these parameters. The data in columns 2, 3, 5, and 6 are identical with that in columns 2, 3, 7, and 8 of Table IV of Ref. 4 since they are all derived from Eq. (3) which is a least-squares fit of the experimental U,u data⁴. Columns 4, 7, 10, and 11 are analogous to, respectively, columns 5, 6, 10, and 11 of Table IV of Ref. 4. The dimensions are: P in megabars, T , T_H in °K, ϵ , $c_{p,H}$, U in cm/ μ sec, and c_p , $c_{p,H}$ in (cal/g)/°K.

Material	P	Shock wave parameters					Isentropic release-wave parameters at $P = 0$					Isentrope parameters				
		V_H/V_0	T_H	c_H	u_H	U	$c_{p,H}$	Z	γ/V_0	T	c	c_p	ϵ	$\gamma(G)/V_0$	V/V_0	T
Ag	0.	1.000	293	0.324	0.	-0.324	0.324	0.	1.000	293	0.324	0.090	1.171	1.036	921	921
	0.1	0.929	332	0.366	0.026	0.378	0.245	0.1275	1.000	298	0.324	0.114	1.153	0.965	951	951
	0.2	0.881	378	0.400	0.048	0.419	0.242	0.2332	1.001	318	0.325	0.137	0.960	0.916	981	981
	0.3	0.845	435	0.430	0.067	0.452	0.282	0.1254	1.003	349	0.325	0.158	0.901	0.878	1011	1011
	0.4	0.817	500	0.457	0.083	0.481	0.333	0.0883	1.005	387	0.326	0.179	0.863	0.848	1041	1041
	0.5	0.794	570	0.481	0.099	0.507	0.389	0.4843	1.008	430	0.327	0.199	0.834	0.823	1071	1071
	0.6	0.775	643	0.504	0.113	0.530	0.450	0.5908	1.010	474	0.328	0.219	0.811	0.801	1101	1101
	0.7	0.758	719	0.526	0.127	0.551	0.515	0.6209	1.013	520	0.329	0.239	0.790	0.782	1131	1131
	0.8	0.744	796	0.546	0.140	0.571	0.584	0.6832	1.016	566	0.329	0.258	0.772	0.765	1162	1162
	0.9	0.731	873	0.565	0.152	0.589	0.657	0.7425	1.018	612	0.330	0.278	0.755	0.749	1192	1192
	1.0	0.720	951	0.583	0.163	0.607	0.734	0.7991	1.021	658	0.331	0.297	0.740	0.735	1222	1222
	1.1	0.710	1028	0.601	0.174	0.623	0.815	0.8533	1.024	704	0.332	0.316	0.722	0.722	1252	1252
	1.2	0.700	1105	0.618	0.185	0.638	0.899	0.9054	1.026	749	0.333	0.335	0.713	0.710	1282	1282
	1.3	0.692	1180	0.634	0.195	0.653	0.988	0.9556	1.029	793	0.334	0.353	0.701	0.699	1312	1312
	1.4	0.684	1255	0.650	0.205	0.667	1.081	1.0042	1.031	836	0.334	0.372	0.690	0.689	1342	1342
	1.5	0.677	1329	0.665	0.215	0.681	1.177	1.0513	1.034	879	0.335	0.391	0.680	0.679	1372	1372
	1.6	0.670	1402	0.680	0.222	0.694	1.278	1.0969	1.036	921	0.336	0.409	0.670	0.670	1402	1402
-																
Au	0.	1.000	293	0.307	-0.	0.307	0.	0.794	1.000	293	0.307	0.062	1.113	1.026	905	905
	0.1	0.953	316	0.352	0.016	0.340	0.166	0.4221	1.006	295	0.308	0.072	1.028	0.979	927	927
	0.2	0.917	343	0.353	0.029	0.366	0.136	0.1492	1.000	304	0.308	0.082	0.985	0.943	948	948
	0.3	0.888	376	0.373	0.042	0.389	0.145	0.2122	1.001	320	0.308	0.091	0.949	0.913	970	970
	0.4	0.864	415	0.391	0.053	0.409	0.161	0.2701	1.002	342	0.308	0.110	0.920	0.888	991	991
	0.5	0.843	459	0.407	0.064	0.427	0.180	0.3259	1.003	369	0.308	0.108	0.894	0.866	1013	1013
	0.6	0.825	507	0.423	0.074	0.444	0.200	0.3744	1.005	398	0.308	0.117	0.872	0.847	1034	1034
	0.7	0.810	559	0.437	0.083	0.459	0.221	0.4221	1.006	431	0.309	0.125	0.852	0.830	1056	1056
	0.8	0.796	613	0.451	0.092	0.473	0.244	0.4675	1.007	465	0.310	0.134	0.834	0.815	1077	1077
	0.9	0.783	669	0.465	0.101	0.487	0.268	0.5108	1.009	500	0.310	0.142	0.817	0.801	1099	1099
	1.0	0.772	727	0.477	0.109	0.500	0.292	0.5524	1.010	536	0.311	0.150	0.802	0.788	1120	1120
	1.1	0.762	786	0.490	0.117	0.512	0.318	0.5924	1.012	573	0.311	0.158	0.788	0.776	1142	1142
	1.2	0.752	846	0.502	0.124	0.523	0.345	0.6309	1.014	610	0.312	0.166	0.775	0.765	1163	1163
	1.3	0.743	907	0.513	0.132	0.534	0.372	0.6682	1.015	647	0.312	0.174	0.763	0.754	1185	1185

U	0.1	343	0.353	0.029	0.366	0.136	0.1492	1.000	304	U.U82	U.U82
U	0.2	376	0.373	0.042	0.389	0.145	0.2122	1.001	320	0.308	0.091
U	0.3	376	0.373	0.053	0.409	0.161	0.2701	1.002	342	0.308	0.100
U	0.4	415	0.391	0.053	0.427	0.180	0.3239	1.003	369	0.309	0.117
U	0.5	459	0.407	0.064	0.444	0.200	0.3744	1.005	398	0.309	0.125
U	0.6	507	0.423	0.074	0.459	0.221	0.4221	1.006	431	0.309	0.134
U	0.7	559	0.437	0.083	0.473	0.244	0.4675	1.007	465	0.310	0.142
U	0.8	613	0.451	0.092	0.487	0.268	0.5108	1.009	500	0.310	0.150
U	0.9	669	0.465	0.101	0.500	0.292	0.5524	1.010	536	0.311	0.150
U	1.0	727	0.477	0.109	0.512	0.318	0.5924	1.012	573	0.311	0.158
U	1.1	786	0.490	0.117	0.523	0.345	0.6309	1.014	610	0.312	0.166
U	1.2	825	0.502	0.124	0.534	0.372	0.6682	1.015	647	0.312	0.174
U	1.3	874	0.513	0.132	0.544	0.400	0.7044	1.017	685	0.313	0.182
U	1.4	907	0.524	0.139	0.554	0.429	0.7394	1.018	722	0.313	0.190
U	1.5	929	0.535	0.146	0.555	0.449	0.7736	1.020	759	0.314	0.198
U	1.6	1020	0.545	0.152	0.564	0.460	0.8068	1.022	796	0.314	0.206
U	1.7	1152	0.556	0.159	0.574	0.491	0.8392	1.023	833	0.315	0.215
U	1.8	1213	0.566	0.165	0.583	0.523	0.8709	1.025	869	0.315	0.221
U	1.9	1274	0.575	0.172	0.592	0.555	0.9018	1.026	905	0.316	0.229
U	2.0	1335	0.585	0.178	0.600	0.589	0.926	1.026	936	0.316	0.235

cd	0.	1.000	293	0.244	0.	0.244	-0-	0-	293	0.244	0.076
cd	0.1	376	0.307	0.038	0.325	0.280	0.2577	1.002	312	0.245	0.121
cd	0.2	475	0.354	0.065	0.377	0.360	0.4477	1.006	362	0.246	0.161
cd	0.3	572	0.392	0.089	0.418	0.483	0.6055	1.012	423	0.247	0.199
cd	0.4	674	0.426	0.109	0.453	0.629	0.7435	1.017	486	0.249	0.236
cd	0.5	722	0.456	0.127	0.482	0.795	0.8673	1.023	548	0.250	0.272
cd	0.6	799	0.484	0.143	0.509	0.980	1.029	608	608	0.251	0.308
cd	0.7	104	0.510	0.159	0.534	1.185	1.0869	1.034	665	0.253	0.344
cd	0.8	112	0.534	0.173	0.556	1.410	1.1859	1.039	721	0.254	0.379
cd	0.9	1112	0.557	0.187	0.577	1.655	1.2795	1.044	774	0.255	0.414
cd	1.0	1301	0.579	0.200	0.597	1.922	1.3683	1.049	824	0.256	0.449
cd	1.1	1390	0.599	0.212	0.616	2.212	1.4531	1.053	812	0.257	0.484
cd	1.2	1475	0.619	0.224	0.634	2.525	1.5344	1.058	919	0.258	0.518
cd	1.3	1531	0.638	0.236	0.651	2.862	1.6125	1.062	963	0.259	0.553
cd	1.4	1538	0.657	0.247	0.667	3.225	1.6879	1.066	1006	0.260	0.587

Co	0.	1.000	294	0.475	0.	0.475	-0-	0-	293	0.475	0.149
Co	0.1	311	0.505	0.022	0.511	0.351	0.0629	1.000	294	0.475	0.169
Co	0.2	333	0.532	0.043	0.541	0.4290	0.1195	1.001	301	0.475	0.187
Co	0.3	360	0.556	0.061	0.567	0.507	0.1713	1.001	314	0.475	0.205
Co	0.4	393	0.579	0.078	0.591	0.532	0.2194	1.001	333	0.476	0.223
Co	0.5	431	0.600	0.094	0.612	0.564	0.2645	1.002	356	0.476	0.239
Co	0.6	474	0.621	0.110	0.631	0.599	0.3071	1.003	383	0.476	0.256
Co	0.7	520	0.640	0.124	0.649	0.635	0.3475	1.004	414	0.477	0.273
Co	0.8	570	0.658	0.138	0.665	0.745	0.3861	1.006	446	0.477	0.293
Co	0.9	623	0.676	0.151	0.681	0.516	0.4230	1.007	480	0.478	0.305
Co	1.0	679	0.693	0.164	0.695	0.558	0.4586	1.008	516	0.479	0.321
Co	1.1	736	0.709	0.176	0.709	0.502	0.4929	1.010	553	0.479	0.336
Co	1.2	794	0.725	0.188	0.722	0.647	0.5260	1.011	591	0.480	0.352
Co	1.3	855	0.740	0.199	0.734	0.693	0.5581	1.012	629	0.481	0.367
Co	1.4	916	0.755	0.210	0.746	0.741	0.5892	1.014	668	0.481	0.383
Co	1.5	977	0.769	0.221	0.757	0.790	0.6195	1.015	707	0.482	0.398
Co	1.6	1040	0.783	0.232	0.768	0.841	0.6490	1.017	746	0.483	0.413

Cr	0.	1.000	293	0.475	0.	0.475	-0-	0-	293	0.475	0.149
Cr	0.1	316	0.559	0.025	0.569	0.154	0.0708	1.000	296	0.522	0.095
Cr	0.2	350	0.591	0.048	0.608	0.141	0.1337	1.000	313	0.522	0.106
Cr	0.3	391	0.621	0.068	0.642	0.158	0.1910	1.001	344	0.522	0.117
Cr	0.4	463	0.649	0.087	0.672	0.178	0.2438	1.002	387	0.523	0.128
Cr	0.5	539	0.675	0.104	0.700	0.200	0.2931	1.003	439	0.523	0.138
Cr	0.6	845	0.704	0.223	0.724	0.223	0.3396	1.004	499	0.524	0.149
Cr	0.7	827	0.721	0.242	0.740	0.242	0.3744	1.005	565	0.524	0.159

0.5	0.843	451	0.600	0.094	0.612	0.364	U-2642	1-UUC	229	0.476	0.256	0.841	0.837	856
0.6	0.323	474	0.621	0.110	0.631	0.399	U-3071	1-003	383	0.476	0.273	0.822	0.819	875
0.7	0.806	520	0.640	0.124	0.649	0.435	U-3475	1-004	414	0.477	0.299	0.804	0.802	893
0.8	0.791	570	0.658	0.138	0.665	0.475	U-3861	1-006	446	0.477	0.305	0.789	0.786	911
0.9	0.776	623	0.676	0.151	0.681	0.516	U-4230	1-007	480	0.478	0.321	0.774	0.772	930
1.0	0.764	679	0.693	0.164	0.695	0.558	U-4586	1-008	516	0.479	0.336	0.760	0.759	948
1.1	0.752	736	0.709	0.176	0.709	0.592	U-4929	1-010	553	0.479	0.352	0.748	0.747	966
1.2	0.741	794	0.725	0.188	0.722	0.647	U-5260	1-011	591	0.481	0.367	0.736	0.735	985
1.3	0.731	855	0.740	0.199	0.734	0.693	U-5581	1-012	629	0.481	0.374	0.724	0.724	1003
1.4	0.721	916	0.755	0.210	0.746	0.741	U-5892	1-014	668	0.481	0.383	0.714	0.714	1022
1.5	0.712	977	0.769	0.221	0.757	0.790	U-6195	1-015	707	0.482	0.398	0.704	0.704	1040
1.6	0.704	1040	0.783	0.232	0.768	0.841	U-6490	1-017	746	0.483	0.413	0.704	0.704	

Cr	0.1	1.000	293	0.522	0.025	0.522	-0.	0.3708	0.	1.000	293	0.522	0.083	1.017
0.2	0.951	316	0.559	0.048	0.608	0.141	0.134	0.3137	1.000	296	0.522	0.095	1.017	1181
0.3	0.891	350	0.591	0.068	0.642	0.158	0.158	0.1910	1.001	313	0.522	0.106	0.942	1213
0.4	0.866	399	0.621	0.087	0.672	0.178	0.178	0.2438	1.002	344	0.522	0.117	0.910	1245
0.5	0.845	539	0.675	0.104	0.700	0.200	0.200	0.2931	1.003	387	0.523	0.128	0.882	1277
0.6	0.821	526	0.699	0.121	0.724	0.223	0.223	0.3396	1.004	439	0.523	0.138	0.859	1308
0.7	0.811	722	0.722	0.137	0.747	0.247	0.247	0.3836	1.005	565	0.524	0.149	0.839	1340
0.8	0.793	825	0.744	0.152	0.769	0.272	0.272	0.4255	1.006	635	0.525	0.169	0.804	1372
0.9	0.783	934	0.765	0.166	0.785	0.299	0.299	0.4656	1.008	709	0.526	0.179	0.790	1404
1.0	0.771	1047	0.785	0.179	0.808	0.325	0.325	0.5041	1.009	784	0.526	0.188	0.777	1436
1.1	0.760	1165	0.804	0.193	0.826	0.353	0.353	0.5311	1.011	862	0.527	0.198	0.766	1468
1.2	0.750	1285	0.823	0.205	0.843	0.382	0.382	0.5769	1.012	941	0.528	0.208	0.754	1499
1.3	0.741	1408	0.841	0.218	0.859	0.411	0.411	0.6116	1.014	1021	0.529	0.217	0.743	1521
1.4	0.732	1533	0.858	0.230	0.875	0.442	0.442	0.6651	1.015	1101	0.530	0.227	0.734	1553
1.5	0.724	1658	0.875	0.241	0.890	0.473	0.473	0.6778	1.017	1181	0.530	0.236	0.724	1585

Cu	0.	1.000	293	0.396	0.	0.396	-0.	0.0978	0.	1.000	293	0.396	0.124	1.026
0.1	0.940	322	0.435	0.026	0.445	0.323	0.445	0.291	0.1817	1.001	309	0.396	0.149	1.026
0.2	0.897	357	0.468	0.048	0.484	0.291	0.468	0.324	0.2564	1.002	332	0.397	0.173	1.027
0.3	0.864	399	0.497	0.068	0.517	0.324	0.497	0.370	0.3243	1.003	361	0.397	0.196	1.027
0.4	0.836	449	0.524	0.086	0.546	0.421	0.421	0.3870	1.005	395	0.398	0.218	0.870	1027
0.5	0.814	505	0.549	0.102	0.571	0.476	0.476	0.4456	1.007	432	0.399	0.239	0.843	902
0.6	0.794	565	0.572	0.118	0.594	0.535	0.535	0.5008	1.009	471	0.399	0.260	0.820	926
0.7	0.777	628	0.594	0.132	0.616	0.575	0.575	0.5531	1.011	511	0.400	0.281	0.799	951
0.8	0.762	694	0.615	0.146	0.635	0.597	0.597	0.6029	1.013	553	0.401	0.302	0.781	976
0.9	0.749	761	0.634	0.159	0.654	0.633	0.633	0.6505	1.015	595	0.402	0.322	0.764	1001
1.0	0.737	829	0.653	0.172	0.671	0.731	0.671	0.6963	1.017	636	0.403	0.342	0.749	1026
1.1	0.726	898	0.671	0.184	0.687	0.802	0.687	0.7403	1.019	678	0.403	0.362	0.734	1051
1.2	0.716	967	0.689	0.196	0.703	0.816	0.703	0.7829	1.021	720	0.404	0.402	0.723	1075
1.3	0.707	1056	0.706	0.207	0.718	0.953	0.718	0.8241	1.023	761	0.405	0.421	0.700	1100
1.4	0.698	1106	0.722	0.218	0.732	1.033	0.732	0.8640	1.026	802	0.406	0.441	0.690	1125
1.5	0.690	1175	0.738	0.228	0.745	1.116	0.745							

Mo	0.	1.000	293	0.516	0.	0.516	-0.	0.0437	0.	1.000	293	0.516	0.077	1.028
0.1	0.966	306	0.538	0.018	0.542	0.118	0.118	0.0842	1.000	301	0.516	0.091	0.955	1061
0.2	0.937	324	0.559	0.035	0.565	0.115	0.115	0.122	1.022	312	0.516	0.097	0.928	1082
0.3	0.912	350	0.579	0.051	0.585	0.132	0.132	0.1577	1.001	338	0.516	0.103	0.904	1123
0.4	0.890	385	0.597	0.066	0.604	0.143	0.143	0.1915	1.001	368	0.516	0.109	0.883	1144
0.5	0.870	427	0.614	0.080	0.621	0.154	0.154	0.2238	1.002	404	0.517	0.115	0.864	1165
0.6	0.852	478	0.631	0.093	0.636	0.166	0.166	0.2546	1.002	446	0.517	0.121	0.847	1185
0.7	0.836	537	0.647	0.106	0.651	0.178	0.178	0.2843	1.003	493	0.517	0.127	0.831	1206
0.8	0.821	602	0.662	0.118	0.664	0.191	0.191	0.3129	1.004	544	0.518	0.133	0.816	1227
0.9	0.808	673	0.677	0.130	0.677	0.204	0.204	0.3405	1.005	598	0.518	0.139	0.802	1248
1.0	0.795	750	0.691	0.142	0.689	0.221	0.221	0.3672	1.005	655	0.519	0.145	0.789	1268
1.1	0.783	831	0.705	0.153	0.700	0.245	0.245	0.3931	1.006	715	0.519	0.150	0.777	1289
1.2	0.772	917	0.718	0.174	0.711	0.271	0.271	0.4183	1.007	777	0.519	0.156	0.766	1310
1.3	0.762	1006	0.731	0.184	0.722	0.304	0.304	0.4428	1.008	841	0.520	0.162	0.755	1331

1.4 0.732 1533 0.858 0.230 0.890 0.473 0.6778 1.017 1181 0.530 0.236 0.725 0.724 0.720

Cu

1.000	293	0.396	0.	-0.396	0.445	0.323	0.0978	1.000	293	0.396	0.124	1.079	1.026	802
0.940	322	0.435	0.026	0.048	0.484	0.291	0.0817	1.001	296	0.396	0.149	1.079	1.026	827
0.897	357	0.468	0.048	0.068	0.517	0.324	0.2564	1.002	332	0.397	0.173	0.943	0.922	852
0.864	399	0.497	0.068	0.086	0.546	0.370	0.3233	1.003	361	0.397	0.196	0.902	0.887	877
0.836	449	0.524	0.102	0.102	0.571	0.421	0.3870	1.005	395	0.399	0.218	0.870	0.859	902
0.814	505	0.549	0.102	0.118	0.594	0.476	0.4456	1.007	432	0.399	0.239	0.843	0.834	926
0.794	565	0.572	0.132	0.132	0.616	0.535	0.5008	1.009	471	0.399	0.281	0.820	0.813	951
0.777	628	0.594	0.146	0.146	0.635	0.597	0.5531	1.011	511	0.400	0.302	0.781	0.777	1001
0.762	694	0.615	0.159	0.159	0.654	0.663	0.6029	1.013	553	0.401	0.322	0.764	0.761	1026
0.749	761	0.634	0.172	0.172	0.671	0.731	0.6505	1.015	595	0.402	0.342	0.749	0.747	1051
0.737	829	0.653	0.184	0.184	0.687	0.802	0.6963	1.017	636	0.403	0.362	0.736	0.734	1075
0.726	898	0.671	0.196	0.196	0.703	0.876	0.7403	1.019	678	0.403	0.382	0.723	0.722	1100
0.716	967	0.689	0.207	0.207	0.718	0.953	0.7829	1.021	720	0.404	0.402	0.711	0.711	1125
0.707	1036	0.706	0.222	0.218	0.732	1.033	0.8241	1.023	761	0.405	0.421	0.700	0.700	1150
0.698	1106	0.722	0.228	0.228	0.745	1.116	0.8640	1.026	802	0.406	0.441	0.690	0.690	1175
0.690	1175	0.738												

Mo

1.000	293	0.516	0.	-0.516	0.542	0.118	0.0437	1.000	293	0.516	0.077	1.028	1.011	1040
0.966	306	0.538	0.018	0.035	0.559	0.115	0.0842	1.000	301	0.516	0.084	0.988	0.977	1061
0.937	324	0.559	0.051	0.051	0.585	0.122	0.1220	1.000	316	0.516	0.097	0.955	0.948	1082
0.912	350	0.579	0.066	0.066	0.604	0.132	0.1577	1.001	338	0.516	0.103	0.928	0.923	1102
0.890	385	0.597	0.080	0.080	0.621	0.143	0.1915	1.001	368	0.516	0.109	0.904	0.901	1123
0.870	427	0.614	0.093	0.093	0.636	0.154	0.2238	1.002	404	0.517	0.115	0.883	0.880	1144
0.852	478	0.631	0.106	0.106	0.651	0.166	0.2546	1.002	446	0.517	0.121	0.864	0.862	1165
0.836	537	0.647	0.118	0.118	0.664	0.178	0.2843	1.003	493	0.517	0.127	0.847	0.845	1185
0.821	602	0.662	0.130	0.130	0.677	0.191	0.3129	1.004	544	0.518	0.133	0.816	0.815	1206
0.808	673	0.677	0.142	0.142	0.689	0.204	0.3405	1.005	598	0.518	0.139	0.802	0.802	1227
0.795	750	0.691	0.153	0.153	0.700	0.218	0.3672	1.005	655	0.519	0.145	0.789	0.789	1248
0.783	831	0.705	0.231	0.231	0.711	0.231	0.3931	1.006	715	0.519	0.150	0.777	0.777	1268
0.772	917	0.718	0.164	0.164	0.722	0.245	0.4183	1.007	777	0.519	0.156	0.766	0.766	1289
0.762	1006	0.731	0.174	0.174	0.731	0.260	0.4428	1.008	841	0.520	0.162	0.755	0.755	1310
0.752	1099	0.744	0.184	0.184	0.744	0.274	0.4667	1.009	906	0.520	0.167	0.745	0.745	1331
0.743	1194	0.756	0.194	0.194	0.756	0.289	0.4901	1.010	973	0.521	0.173	0.735	0.735	1351
0.734	1293	0.768	0.204	0.204	0.770	0.305	0.5128	1.011	1040	0.522	0.178	0.726	0.726	1372
0.726	1393	0.780												1393
0.726	1393	0.780												

Ni

1.000	293	0.465	0.	-0.465	0.506	0.023	0.344	1.000	293	0.465	0.139	1.045	1.018	759	
0.954	314	0.497	0.043	0.043	0.541	0.286	0.1333	1.000	302	0.465	0.160	0.992	0.973	779	
0.919	338	0.527	0.052	0.052	0.553	0.061	0.303	1.001	317	0.465	0.179	0.950	0.937	798	
0.889	368	0.553	0.061	0.061	0.571	0.078	0.333	1.002	338	0.465	0.197	0.917	0.907	818	
0.865	404	0.578	0.078	0.078	0.597	0.098	0.333	0.2923	1.003	363	0.466	0.215	0.890	0.881	838
0.843	445	0.600	0.094	0.094	0.621	0.121	0.368	0.407	392	0.466	0.233	0.866	0.859	857	
0.825	491	0.622	0.109	0.109	0.643	0.147	0.3837	1.004	424	0.467	0.267	0.846	0.839	877	
0.809	541	0.642	0.123	0.123	0.663	0.174	0.3826	1.005	457	0.468	0.284	0.826	0.822	897	
0.794	594	0.662	0.136	0.136	0.682	0.194	0.4244	1.006	493	0.468	0.301	0.794	0.791	916	
0.781	649	0.680	0.149	0.149	0.699	0.217	0.4644	1.008	529	0.469	0.317	0.779	0.778	936	
0.768	706	0.698	0.162	0.162	0.716	0.241	0.581	1.009	566	0.470	0.334	0.766	0.765	956	
0.757	765	0.715	0.174	0.174	0.732	0.259	0.598	1.011	604	0.470	0.350	0.753	0.753	975	
0.747	825	0.732	0.185	0.185	0.747	0.274	0.679	0.5755	643	0.471	0.366	0.743	0.742	1015	
0.738	887	0.748	0.196	0.196	0.761	0.295	0.784	0.6436	681	0.472	0.382	0.732	0.732	1034	
0.729	948	0.764	0.207	0.207	0.775	0.317	0.788	0.938	720	0.472	0.398	0.722	0.722	1054	
0.721	1011	0.779	0.217	0.217	0.788	0.330	0.793	0.894	759	0.473	0.413	0.713	0.713	1074	
0.713	1074														

TABLE II. Summary of calculations and results.

Material	P	Shock wave parameters				Isentropic release-wave parameters at P_0				Isentrope parameters						
		V_H/V_0	T_H	c_H	u_H	$c_{p,H}$	U	c_p	γ	V/V_0	T	c	c_p	$\gamma(a)/\gamma_0$	V/V_0	T
Pb	0.	1.000	293	0.203	0.	0.203	-0.	0.	1.000	293	0.203	0.055	1.226	1.059	964	
	0.1	0.865	380	0.255	0.035	0.266	0.183	0.2585	1.002	315	0.203	0.087	1.020	0.921	1016	
	0.2	0.796	490	0.294	0.060	0.246	0.490	0.490	1.007	374	0.204	0.117	0.913	0.848	1068	
	0.3	0.751	612	0.326	0.081	0.337	0.335	0.6072	1.013	444	0.205	0.144	0.842	0.797	1120	
	0.4	0.718	738	0.354	0.100	0.363	0.439	0.7454	1.019	517	0.207	0.171	0.790	0.758	1172	
	0.5	0.693	863	0.379	0.116	0.385	0.556	0.8698	1.026	589	0.208	0.198	0.749	0.727	1224	
	0.6	0.673	985	0.402	0.132	0.405	0.687	0.9838	1.032	658	0.209	0.224	0.716	0.700	1276	
	0.7	0.656	1103	0.424	0.146	0.423	0.832	1.0896	1.037	725	0.210	0.250	0.688	0.678	1328	
	0.8	0.642	1218	0.444	0.159	0.440	0.991	1.1888	1.043	789	0.212	0.275	0.658	0.658	1380	
	0.9	0.630	1328	0.463	0.171	0.455	1.165	1.2825	1.049	850	0.213	0.301	0.643	0.640	1432	
Sn	1.0	0.619	1434	0.481	0.183	0.470	1.354	1.3715	1.054	908	0.214	0.327	0.625	0.624	1484	
	1.1	0.609	1536	0.498	0.195	0.483	1.558	1.4565	1.059	964	0.215	0.352	0.609	0.609	1536	
	1.2	0.601	1635	0.515	0.206	0.496	1.780	1.5379	1.064	1018	0.216	0.378	0.588	0.588	1588	
	1.3	0.593	1730	0.531	0.216	0.509	2.018	1.6162	1.068	1069	0.217	0.403	0.563	0.563	1640	
	1.4	0.586	1821	0.546	0.226	0.521	2.275	1.6917	1.073	1118	0.218	0.428	0.543	0.543	1692	
	0.	1.000	293	0.264	0.	0.264	-0.	0.	1.000	293	0.264	0.068	1.191	1.050	1100	
	0.1	0.871	380	0.326	0.042	0.338	0.182	0.2355	1.002	319	0.264	0.103	0.990	0.919	1161	
	0.2	0.802	505	0.373	0.074	0.386	0.252	0.4120	1.006	391	0.266	0.136	0.874	0.846	1223	
	0.3	0.757	653	0.412	0.100	0.423	0.342	0.5596	1.011	480	0.267	0.167	0.812	0.795	1285	
	0.4	0.724	809	0.446	0.123	0.454	0.445	0.6890	1.017	575	0.269	0.196	0.768	0.756	1347	
Th	0.5	0.698	968	0.477	0.144	0.481	0.560	0.8056	1.023	669	0.270	0.226	0.733	0.724	1408	
	0.6	0.677	1125	0.505	0.163	0.505	0.687	0.9126	1.029	762	0.272	0.255	0.703	0.698	1470	
	0.7	0.659	1280	0.531	0.181	0.526	0.926	1.0121	1.034	851	0.273	0.283	0.678	0.675	1532	
	0.8	0.644	1430	0.556	0.198	0.546	0.978	1.054	1.039	937	0.274	0.312	0.654	0.654	1593	
	0.9	0.631	1576	0.579	0.213	0.565	1.143	1.1936	1.045	1020	0.276	0.340	0.638	0.636	1655	
	1.0	0.620	1717	0.601	0.228	0.582	1.321	1.2774	1.050	1100	0.277	0.368	0.620	0.620	1717	
	1.1	0.610	1854	0.622	0.243	0.598	1.512	1.3574	1.054	1176	0.278	0.397	0.600	0.600	1779	
	1.2	0.601	1986	0.643	0.257	0.614	1.717	1.4341	1.059	1249	0.280	0.425	0.571	0.571	1841	
	1.3	0.593	2114	0.662	0.270	0.628	1.937	1.5079	1.064	1320	0.281	0.453	0.542	0.542	1893	
	1.4	0.585	2238	0.681	0.282	0.642	2.172	1.5791	1.068	1388	0.282	0.481	0.513	0.513	1951	
Pb	0.	1.000	293	0.213	0.	0.213	-0.	0.	1.000	293	0.213	0.028	1.242	1.070	2336	
	0.1	0.869	385	0.256	0.033	0.260	0.059	0.2005	1.001	330	0.213	0.041	1.039	0.938	2428	
	0.2	0.795	555	0.289	0.059	0.290	0.083	0.2552	1.005	439	0.214	0.052	0.914	0.860	2521	
	0.3	0.744	775	0.317	0.081	0.313	0.111	0.4860	1.010	584	0.215	0.063	0.832	0.805	2614	
	0.4	0.706	1022	0.341	0.100	0.332	0.142	0.6013	1.015	743	0.216	0.074	0.774	0.761	2706	
	0.5	0.676	1281	0.364	0.118	0.348	0.177	0.7057	1.021	906	0.218	0.084	0.730	0.726	2799	
	0.6	0.652	1544	0.384	0.134	0.362	0.215	0.8017	1.026	1068	0.219	0.094	0.694	0.696	2891	
	0.7	0.631	1806	0.403	0.149	0.374	0.256	0.8911	1.032	1227	0.220	0.104	0.665	0.670	2984	
	0.8	0.614	2026	0.421	0.163	0.386	0.300	0.9750	1.037	1382	0.221	0.114	0.640	0.640	3076	
	0.9	0.598	2320	0.438	0.176	0.397	0.348	1.0545	1.042	1533	0.222	0.124	0.619	0.627	3169	
Sn	1.0	0.585	2569	0.454	0.189	0.407	0.399	1.1301	1.047	1678	0.223	0.134	0.601	0.608	3262	
	1.1	0.573	2812	0.470	0.201	0.416	0.453	1.2023	1.052	1819	0.224	0.144	0.585	0.591	3354	
	1.2	0.562	3049	0.484	0.212	0.425	0.511	1.2716	1.057	1955	0.225	0.154	0.570	0.575	3447	
	1.3	0.552	3280	0.499	0.223	0.433	0.573	1.3383	1.062	2086	0.226	0.164	0.558	0.561	3539	
	1.4	0.543	3505	0.512	0.234	0.441	0.638	1.4027	1.066	2213	0.227	0.174	0.546	0.548	3632	
	1.5	0.535	3724	0.526	0.244	0.448	0.707	1.4649	1.070	2336	0.228	0.184	0.535	0.535	3724	

-

1.4 0.585 2238 0.681 0.282 0.642 2.172 1.5791 1.068 1.388 0.282 0.401

Th 0.1 1.000 293 0.213 0.033 0.260 0.059 0.2005 1.000 293 0.213 0.028 2336
 0.869 385 0.256 0.289 0.059 0.290 0.083 0.5552 1.001 330 0.213 0.041 0.938 2428
 0.2 0.795 555 0.317 0.081 0.313 0.111 0.4860 1.010 584 0.215 0.052 0.914 2521
 0.3 0.744 775 0.341 0.100 0.332 0.142 0.6013 1.015 743 0.216 0.063 0.832 2614
 0.4 0.706 1022 0.341 0.100 0.332 0.118 0.348 0.177 0.7057 1.021 906 0.218 0.074 0.805 2706
 0.5 0.676 1281 0.364 0.118 0.362 0.215 0.8017 1.026 1068 0.219 0.084 0.730 2799
 0.6 0.652 1544 0.384 0.134 0.362 0.256 0.8911 1.032 1227 0.220 0.104 0.665 0.726 2891
 0.7 0.631 1806 0.403 0.149 0.374 0.256 0.8911 1.032 1382 0.221 0.114 0.640 0.696 2984
 0.8 0.614 2365 0.421 0.163 0.386 0.300 0.9750 1.037 1533 0.222 0.124 0.619 0.647 3076
 0.9 0.598 2320 0.438 0.176 0.397 0.348 1.0545 1.042 1678 0.223 0.134 0.601 0.627 3169
 1.0 0.585 2569 0.454 0.189 0.407 0.399 1.301 1.047 1819 0.224 0.144 0.585 0.608 3262
 1.1 0.573 2812 0.470 0.201 0.416 0.453 1.2023 1.052 1955 0.225 0.154 0.570 0.591 3354
 1.2 0.562 3049 0.484 0.212 0.421 0.511 1.2716 1.057 2086 0.226 0.164 0.558 0.561 3447
 1.3 0.552 3280 0.499 0.223 0.433 0.573 1.3383 1.062 2213 0.227 0.174 0.546 0.548 3539
 1.4 0.543 3505 0.512 0.234 0.441 0.638 1.4027 1.066 2336 0.228 0.184 0.535 0.535 3632
 1.5 0.535 3724 0.526 0.244 0.448 0.707 1.4649 1.070 2336 0.228 0.184 0.535 0.535 3724

T1 0.1 1.000 293 0.478 0.042 0.524 0.208 0.964 1.000 293 0.478 0.128 1.023
 0.919 327 0.524 0.042 0.524 0.239 0.793 1.001 301 0.478 0.154 0.956 1189
 0.2 0.660 386 0.564 0.079 0.559 0.239 1.073 1.003 335 0.478 0.178 0.893 1233
 0.3 0.615 473 0.599 0.111 0.586 0.283 1.2531 1.003 394 0.479 0.201 0.843 1277
 0.4 0.777 534 0.631 0.141 0.609 0.331 1.3203 1.005 470 0.480 0.224 0.801 1321
 0.5 0.746 713 0.661 0.168 0.629 0.383 1.3824 1.007 558 0.481 0.246 0.762 1365
 0.6 0.719 955 0.688 0.193 0.646 0.438 1.4404 1.009 655 0.482 0.268 0.734 1409
 0.7 0.696 1007 0.714 0.217 0.662 0.496 1.4950 1.012 757 0.484 0.289 0.707 1447
 0.8 0.675 1167 0.739 0.240 0.676 0.556 1.5468 1.015 862 0.485 0.310 0.684 1541
 0.9 0.657 1333 0.763 0.262 0.688 0.620 1.5961 1.017 970 0.486 0.331 0.662 1586
 1.0 0.641 1502 0.785 0.282 0.700 0.686 1.6433 1.020 1079 0.488 0.352 0.643 1630
 1.1 0.626 1674 0.807 0.302 0.711 0.755 0.6887 1.023 1189 0.489 0.373 0.625 1674

T1 0.1 1.000 293 0.186 0.035 0.250 0.260 0.2876 1.000 293 0.186 0.062 1.241
 0.853 385 0.239 0.035 0.289 0.342 0.4952 1.003 315 0.186 0.103 0.905 813
 0.2 0.781 489 0.278 0.061 0.310 0.382 0.6665 1.016 428 0.189 0.140 0.828 862
 0.3 0.736 600 0.310 0.082 0.319 0.468 1.023 490 0.190 0.175 0.795 911
 0.4 0.703 710 0.338 0.101 0.344 0.619 0.8157 1.023 550 0.191 0.209 0.750 961
 0.5 0.679 818 0.362 0.117 0.366 0.793 0.9996 1.030 550 0.191 0.243 0.713 1010
 0.6 0.658 922 0.385 0.132 0.385 0.989 1.0721 1.037 607 0.193 0.277 0.683 1060
 0.7 0.642 1021 0.406 0.146 0.402 1.208 1.1858 1.043 662 0.194 0.310 0.657 1109
 0.8 0.628 1117 0.426 0.159 0.418 1.452 1.2922 1.049 714 0.195 0.343 0.635 1158
 0.9 0.616 1208 0.445 0.171 0.433 1.721 1.3927 1.055 763 0.196 0.375 0.616 1208
 1.0 0.605 1295 0.463 0.183 0.447 2.017 1.4881 1.061 810 0.197 0.408 0.595 1258
 1.1 0.596 1378 0.479 0.194 0.461 2.340 1.5791 1.066 855 0.198 0.441 0.579 1303
 1.2 0.587 1458 0.496 0.204 0.473 2.694 1.6664 1.071 897 0.199 0.473 0.559 1352
 1.3 0.580 1535 0.511 0.215 0.485 3.080 1.7502 1.076 938 0.200 0.506 0.538 1402
 1.4 0.573 1609 0.526 0.225 0.497 3.499 1.8310 1.081 977 0.201 0.538 0.518 1451
 1.5 0.567 1680 0.541 0.234 0.508 3.954 1.9091 1.086 1014 0.202 0.538 0.517 1500

T1 0.1 1.000 293 0.511 0.030 0.551 0.198 0.0710 1.000 293 0.511 0.121 1.017
 0.945 316 0.547 0.030 0.584 0.207 0.134 0.0710 1.000 296 0.511 0.138 0.962 1024
 0.2 0.902 349 0.579 0.057 0.584 0.232 0.1914 1.001 312 0.511 0.155 0.926 1053
 0.3 0.867 397 0.609 0.081 0.613 0.232 0.2444 1.002 342 0.511 0.171 0.883 1082
 0.4 0.838 459 0.636 0.103 0.637 0.262 0.2938 1.003 384 0.513 0.187 0.859 1111
 0.5 0.812 533 0.661 0.124 0.659 0.294 0.2938 1.003 435 0.513 0.202 0.829 1140
 0.533 0.661 0.124 0.659 0.294 0.2938 1.003 435 0.513 0.202 0.802 1169

V 0.1 1.000 293 0.511 0.030 0.551 0.198 0.0710 1.000 293 0.511 0.121 1.017
 0.945 316 0.547 0.030 0.584 0.207 0.134 0.0710 1.000 296 0.511 0.138 0.962 1024
 0.2 0.902 349 0.579 0.057 0.584 0.232 0.1914 1.001 312 0.511 0.155 0.926 1053
 0.3 0.867 397 0.609 0.081 0.613 0.232 0.2444 1.002 342 0.511 0.171 0.883 1082
 0.4 0.838 459 0.636 0.103 0.637 0.262 0.2938 1.003 384 0.513 0.187 0.859 1111
 0.5 0.812 533 0.661 0.124 0.659 0.294 0.2938 1.003 435 0.513 0.202 0.802 1169

(3)

0.2	0.660	386	0.204	0.599	0.111	0.586	0.283	0.2531	1.003	394	0.479	0.201	0.843	0.835	1321
0.3	0.615	473	0.631	0.141	0.609	0.331	0.3203	1.005	470	0.480	0.224	0.801	0.796	1365	
0.4	0.777	584	0.661	0.168	0.629	0.383	0.3824	1.007	558	0.481	0.246	0.765	0.762	1409	
0.5	0.746	713	0.688	0.193	0.646	0.438	0.4404	1.009	655	0.482	0.268	0.734	0.733	1453	
0.6	0.719	855	0.714	0.217	0.662	0.496	0.4950	1.012	757	0.484	0.289	0.707	0.707	1497	
0.7	0.696	1007	0.739	0.240	0.676	0.556	0.5468	1.015	862	0.485	0.310	0.684	0.684	1541	
0.8	0.675	1167	0.763	0.262	0.688	0.620	0.5961	1.017	970	0.486	0.331	0.662	0.663	1586	
0.9	0.657	1333	0.785	0.282	0.700	0.689	0.6433	1.020	1079	0.488	0.352	0.643	0.643	1630	
1.0	0.641	1502	0.807	0.302	0.711	0.755	0.6887	1.023	1189	0.489	0.373	0.625	0.626	1674	

T1	0.	293	0.186	0.	0.186	-0.	0.	0.2876	1.003	293	0.186	0.062	1.241	1.055	763
0.1	0.853	385	0.239	0.035	0.250	0.260	0.4952	1.009	315	0.186	0.103	0.979	0.905	813	
0.2	0.781	489	0.278	0.061	0.289	0.342	0.6665	1.016	428	0.187	0.140	0.859	0.828	862	
0.3	0.736	600	0.310	0.082	0.319	0.468	0.8157	1.023	490	0.190	0.209	0.795	0.776	911	
0.4	0.703	710	0.338	0.100	0.344	0.619	0.9496	1.030	550	0.191	0.243	0.750	0.736	961	
0.5	0.679	818	0.362	0.117	0.366	0.793	1.0721	1.037	607	0.193	0.277	0.713	0.704	1010	
0.6	0.658	922	0.386	0.132	0.385	0.989	1.208	1.043	662	0.194	0.310	0.683	0.677	1060	
0.7	0.642	1021	0.406	0.146	0.402	1.058	1.2922	1.049	714	0.195	0.343	0.657	0.654	1109	
0.8	0.628	1117	0.426	0.159	0.418	1.452	1.3927	1.055	763	0.196	0.375	0.635	0.634	1158	
0.9	0.616	1208	0.445	0.171	0.433	1.721	1.4881	1.061	810	0.197	0.408	0.616	0.616	1208	
1.0	0.605	1295	0.463	0.183	0.447	2.017	1.5791	1.066	855	0.198	0.441	0.598	0.598	1341	
1.1	0.596	1378	0.479	0.194	0.461	2.340	1.6664	1.071	897	0.199	0.473	0.578	0.578	1482	
1.2	0.587	1458	0.496	0.204	0.473	2.694	1.7502	1.076	938	0.200	0.506	0.558	0.558	1623	
1.3	0.580	1535	0.511	0.215	0.485	3.080	1.8310	1.081	977	0.201	0.538	0.506	0.506	1764	
1.4	0.573	1609	0.526	0.225	0.497	3.499	1.8801	1.086	1014	0.202	0.571	0.478	0.478	1905	
1.5	0.567	1680	0.541	0.234	0.508	3.954	1.9091	1.091							

V	0.	293	0.511	0.	0.511	-0.	0.	1.000	1.000	293	0.511	0.121	1.035	1.017	995
0.1	0.945	316	0.547	0.030	0.551	0.198	0.0710	1.001	312	0.511	0.138	0.972	0.962	1024	
0.2	0.902	349	0.579	0.057	0.584	0.207	0.1341	1.000	384	0.511	0.153	0.926	0.918	1053	
0.3	0.867	397	0.609	0.081	0.613	0.232	0.1914	1.001	432	0.511	0.171	0.889	0.883	1082	
0.4	0.838	459	0.636	0.103	0.637	0.262	0.2444	1.002	484	0.511	0.187	0.852	0.852	1111	
0.5	0.812	533	0.661	0.124	0.659	0.294	0.2938	1.003	535	0.513	0.202	0.829	0.826	1140	
0.6	0.790	618	0.685	0.144	0.678	0.328	0.3403	1.005	493	0.513	0.218	0.804	0.802	1169	
0.7	0.771	711	0.707	0.162	0.696	0.363	0.3844	1.006	556	0.514	0.232	0.782	0.781	1199	
0.8	0.753	810	0.729	0.180	0.713	0.400	0.4264	1.008	624	0.515	0.247	0.762	0.762	1228	
0.9	0.737	916	0.749	0.197	0.728	0.439	0.4666	1.009	694	0.516	0.266	0.744	0.744	1257	
1.0	0.723	1025	0.769	0.213	0.742	0.478	0.5051	1.011	768	0.517	0.276	0.728	0.728	1286	
1.1	0.709	1139	0.788	0.229	0.755	0.519	0.5422	1.013	842	0.517	0.290	0.713	0.713	1315	
1.2	0.697	1255	0.806	0.244	0.768	0.562	0.5781	1.015	918	0.518	0.305	0.699	0.699	1344	
1.3	0.686	1373	0.824	0.259	0.780	0.605	0.6128	1.017	995	0.519	0.319	0.686	0.686	1373	

W	0.	293	0.400	0.	0.400	-0.	0.	1.000	1.000	293	0.400	0.053	1.027	1.013	1036
0.1	0.970	305	0.416	0.024	0.431	0.081	0.0766	1.000	298	0.401	0.061	0.957	0.957	1072	
0.2	0.944	319	0.445	0.035	0.451	0.084	0.1113	1.000	308	0.401	0.065	0.941	0.934	1089	
0.3	0.921	339	0.471	0.055	0.477	0.095	0.1442	1.001	323	0.401	0.069	0.919	0.913	1107	
0.4	0.901	364	0.488	0.046	0.465	0.089	0.1754	1.001	343	0.401	0.073	0.900	0.900	1125	
0.5	0.882	395	0.483	0.071	0.488	0.102	0.2053	1.001	368	0.401	0.076	0.882	0.882	1142	
0.6	0.866	431	0.494	0.074	0.500	0.109	0.2339	1.002	398	0.401	0.080	0.865	0.862	1160	
0.7	0.850	473	0.505	0.083	0.510	0.117	0.2615	1.002	431	0.401	0.084	0.850	0.847	1177	
0.8	0.837	520	0.516	0.091	0.520	0.124	0.2881	1.003	467	0.402	0.088	0.836	0.834	1195	
0.9	0.824	571	0.526	0.099	0.529	0.132	0.3139	1.004	505	0.402	0.091	0.823	0.823	1213	
1.0	0.812	626	0.536	0.107	0.538	0.140	0.3388	1.004	547	0.402	0.095	0.810	0.809	1230	
1.1	0.800	685	0.546	0.115	0.548	0.148	0.3630	1.005	590	0.403	0.098	0.799	0.798	1248	
1.2	0.790	747	0.546	0.115	0.554	0.157	0.3866	1.006	635	0.403	0.102	0.788	0.787	1266	
1.3	0.780	812	0.555	0.122	0.562	0.166	0.4096	1.007	682	0.403	0.105	0.778	0.777	1283	
1.4	0.771	880	0.565	0.129	0.569	0.174	0.4320	1.008	730	0.404	0.109	0.768	0.767	1301	
1.5	0.762	950	0.574	0.136	0.576	0.182	0.4538	1.008	779	0.404	0.112	0.758	0.758	1319	

		W	0.259	0.280	0.605	0.6128	1.017	995	0.519	0.319	0.686	0.686	1373
1.3	0.686	1373	0.824	0.400	0.400	0.419	0.090	0.397	1.000	1.000	0.533	1.027	1.036
0.1	0.970	305	0.416	0.013	0.024	0.436	0.081	0.0766	1.000	2.98	0.401	0.995	1054
0.2	0.944	319	0.431	0.035	0.035	0.451	0.084	0.1113	1.000	3.08	0.401	0.957	1072
0.3	0.921	339	0.445	0.046	0.046	0.465	0.089	0.1462	1.001	3.23	0.401	0.941	1089
0.4	0.901	364	0.458	0.055	0.055	0.477	0.095	0.1754	1.001	3.43	0.401	0.934	1107
0.5	0.882	395	0.471	0.065	0.065	0.489	0.102	0.2053	1.001	3.68	0.401	0.919	1124
0.6	0.866	431	0.483	0.074	0.074	0.500	0.109	0.2339	1.002	3.98	0.401	0.900	1142
0.7	0.850	473	0.494	0.083	0.083	0.510	0.117	0.2615	1.002	4.31	0.401	0.882	1160
0.8	0.837	520	0.505	0.091	0.091	0.520	0.124	0.2881	1.003	4.67	0.402	0.862	1177
0.9	0.824	571	0.516	0.099	0.099	0.529	0.132	0.3139	1.004	5.05	0.402	0.847	1195
1.0	0.812	626	0.526	0.107	0.107	0.538	0.140	0.3388	1.004	5.47	0.402	0.834	1213
1.1	0.800	685	0.536	0.115	0.115	0.546	0.148	0.3630	1.005	5.9	0.402	0.823	1230
1.2	0.790	747	0.546	0.122	0.122	0.554	0.157	0.3866	1.006	6.35	0.403	0.810	1248
1.3	0.780	812	0.555	0.129	0.129	0.562	0.166	0.4096	1.007	6.82	0.403	0.805	1266
1.4	0.771	880	0.565	0.136	0.136	0.569	0.174	0.4320	1.008	7.3	0.404	0.808	1283
1.5	0.762	950	0.574	0.143	0.143	0.576	0.183	0.4538	1.008	7.79	0.404	0.809	1301
1.6	0.754	1022	0.582	0.150	0.150	0.583	0.192	0.4752	1.009	8.29	0.404	0.812	1319
1.7	0.746	1096	0.591	0.157	0.157	0.589	0.202	0.4961	1.010	8.8	0.405	0.816	1336
1.8	0.738	1172	0.599	0.163	0.163	0.596	0.211	0.5166	1.011	9.31	0.405	0.820	1354
1.9	0.731	1249	0.607	0.170	0.170	0.602	0.221	0.5367	1.012	9.84	0.405	0.824	1372
2.0	0.725	1327	0.615	0.176	0.176	0.608	0.231	0.5564	1.013	10.36	0.406	0.828	1389
2.1	0.718	1407	0.623										1407

24

	Zn	0.	0.305	-0.	0.	0.305	0.	0.305	0.	0.305	0.	0.305	0.	0.45
1.000	0.895	293	0.365	0.038	0.380	0.576	0.1962	1.001	302	0.305	0.143	1.180	737	
0.1	0.834	352	0.411	0.068	0.432	0.586	0.3482	1.004	331	0.306	0.204	0.939	772	
0.2	0.793	416	0.450	0.093	0.472	0.723	0.4768	1.008	369	0.307	0.260	0.916	806	
0.3	0.762	488	0.485	0.115	0.507	0.893	0.5904	1.012	411	0.309	0.313	0.851	840	
0.4	0.737	564	0.516	0.136	0.537	1.086	0.6931	1.016	453	0.309	0.365	0.808	874	
0.5	0.717	641	0.545	0.154	0.564	1.299	0.7879	1.021	498	0.310	0.415	0.776	908	
0.6	0.701	718	0.572	0.171	0.588	1.533	0.8759	1.025	541	0.311	0.465	0.742	942	
0.7	0.686	794	0.597	0.188	0.610	1.786	0.9587	1.029	583	0.313	0.514	0.726	977	
0.8	0.673	868	0.621	0.203	0.631	2.058	1.0370	1.034	623	0.314	0.563	0.706	1011	
0.9	0.662	941	0.644	0.217	0.651	2.351	1.1116	1.038	662	0.315	0.611	0.687	1045	
1.0	0.652	1011	0.666	0.231	0.670	2.664	1.1828	1.042	701	0.316	0.660	0.671	1079	
1.1	0.643	1080	0.687	0.245	0.687	2.998	1.2512	1.045	737	0.318	0.708	0.657	1113	
1.2	0.635	1148	0.707	0.258	0.704	3.254	1.3170	1.049	773	0.319	0.755	0.643	1148	
1.3	0.628	1213	0.726	0.270	0.720	3.732	1.3804	1.053	808	0.320	0.803			
1.4	0.621	1276	0.745	0.282	0.736	4.133	1.4419	1.056	841	0.321	0.851			
1.5	0.621	1338												

^aThese data were obtained from Dr. R. G. McQueen. They represent the numerical values from which the rightmost isentropes in Figs. 10 to 25 of Ref. 4 were plotted.

4

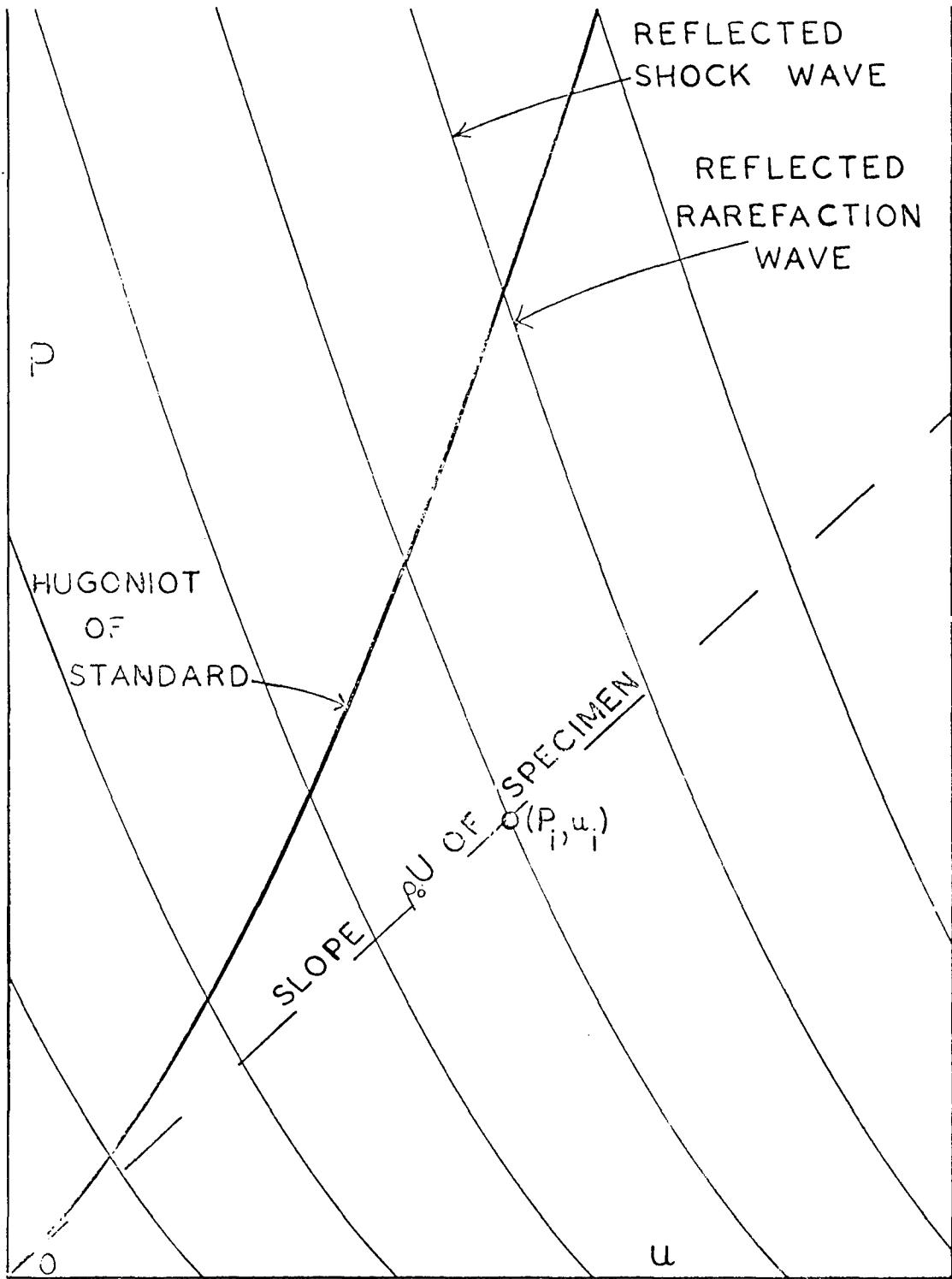
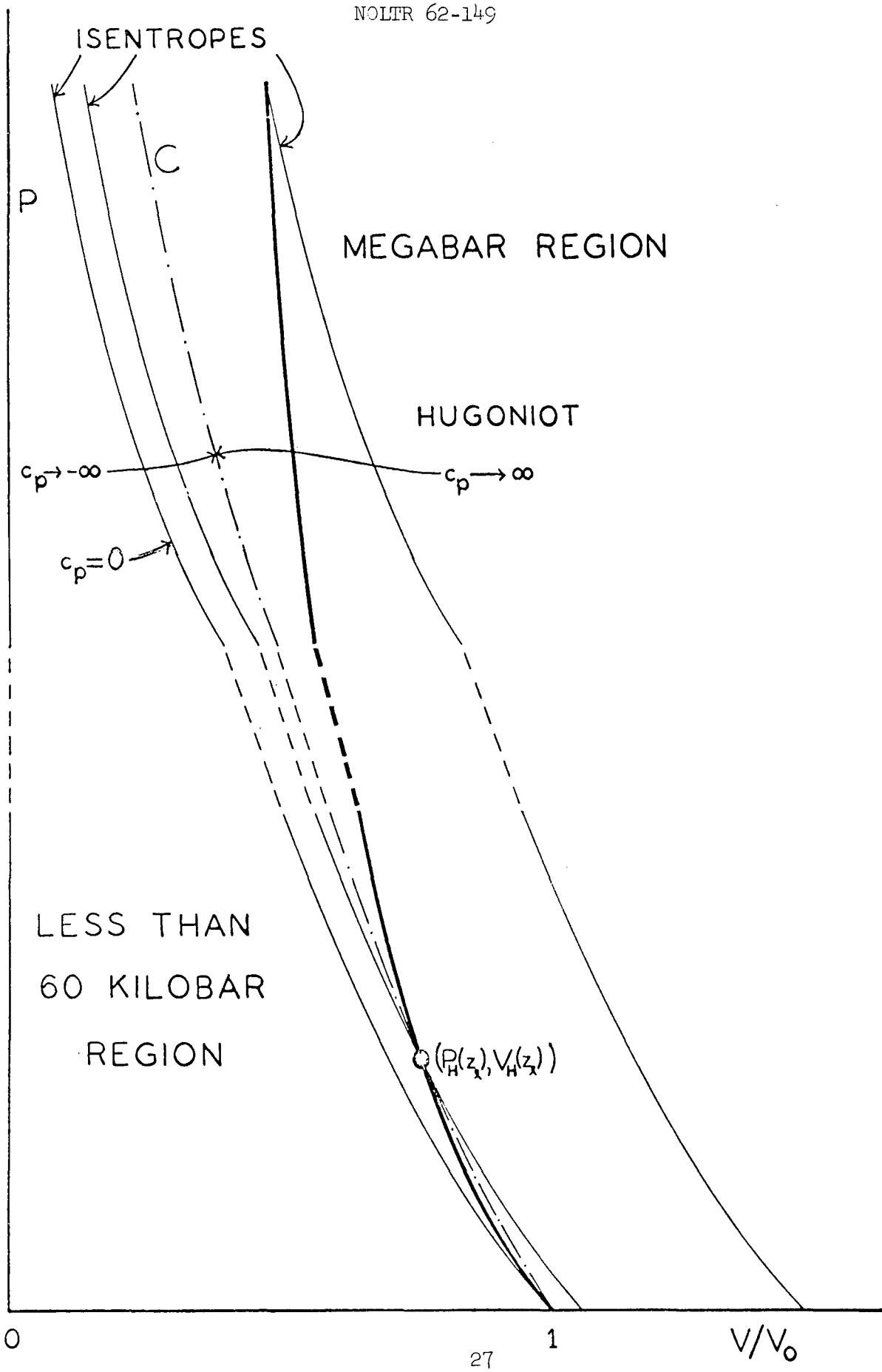


Fig. 1. Pressure versus particle velocity curves for an initial shock in a metal standard followed by a reflected shock or rarefaction wave, and a typical graphical solution to determine P_1, u_1 for a test specimen.

Fig. 2. Pressure versus volume curves for a metal that satisfies the mirror-image assumption. The dash-dot curve, C, is defined by Eq. (50) and is the locus of points on which $c_p \rightarrow \pm \infty$ depending upon the direction of approach. The intersection of C with Hugoniot occurs at $P_H(z_\lambda)$, $V_H(z_\lambda)$ where $P_H(z_\lambda)$ lies between 10 to 50 kilobars for the 16 metals considered. The figure is split into a megabar region and a "less than 60 kilobar region".



DISTRIBUTION

	<u>No. of Copies</u>
SPIA (Distribution List)	100
Chief, Bureau of Naval Weapons	
Washington 25, D.C.	
Attn: Library (DLI-3).....	1
Attn: SP-271.....	4
Attn: SP-20.....	1
Attn: SP-27.....	1
Attn: RMP.....	3
Attn: F-12.....	1
Attn: RUME-32, E. M. Fisher.....	1
Attn: RUME-33, G. D. Edwards.....	1
Office of Naval Research, Washington 25, D.C.	
Attn: Power Branch (Code 429).....	2
Aerojet-General Corporation	
1711 Woodruff Avenue, Downey, California	
Attn: Dr. L. Zernow.....	3
Aerojet-General Corporation	
P.O. Box 1168, Sacramento, California	
Attn: Dr. W. Kirchner.....	4
Lockheed Missiles and Space Company	
A Division of Lockheed Aircraft Corporation	
1122 Jagels Road, Sunnyvale, California	
Attn: Mr. J. Lightfoot.....	3
Bureau of Naval Weapons Representative	
(Special Projects Office)	
Lockheed Missiles and Space Company	
P.O. Box 504, Sunnyvale, California	
Attn: SPL-313.....	2
Bureau of Naval Weapons Resident Representative	
(Special Projects Office)	
Aerojet-General Corporation	
Sacramento, California	
Attn: SPLA-30.....	2
Bureau of Naval Weapons Branch Representative	
Allegany Ballistics Laboratory	
Cumberland, Maryland	
Attn: SPH-30.....	2
Allegany Ballistics Laboratory	
Hercules Powder Company	
Cumberland, Maryland	
Attn: Dr. N. F. LeBlanc.....	
Mr. R. Richardson	4

DISTRIBUTION (Cont'd.)

	<u>No. of Copies</u>
Aeronutronics A Division of Ford Motor Company Ford Road, Newport Beach, California Attn: Mr. S. Weller..... Mr. M. Boyer	3
Rohm and Haas Company Redstone Arsenal Huntsville, Alabama Attn: Dr. H. Shuey.....	3
Commander, U.S. Naval Ordnance Test Station China Lake, California Attn: Code 453	2
Attn: Code 5008	1
Armed Services Explosives Safety Board Building T-7, Gravelly Point Washington, D.C. Attn: Mr. R. C. Herman	1
Stanford Research Institute Liquid Propellant Department Propulsion Sciences Division Menlo Park, California Attn: Dr. A. B. Amster	3
U.S. Bureau of Mines 4800 Forbes Street, Pittsburgh 13, Pennsylvania Attn: Dr. C. M. Mason	1
Director, Office of the Secretary of Defense Adv. Res. Proj. Agency, Washington 25, D.C. Attn: Dr. John F. Kincaid.....	1
University of Cal. Lawrence Radiation Lab. P.O. Box 808, Livermore, California Attn: Dr. G. Dorrough	1
Los Alamos Scientific Laboratory P.O. Box 1663, Los Alamos, New Mexico Attn: Dr. L. C. Smith	1
Liquid Propellant Information Agency The Johns Hopkins University Applied Physics Laboratory 8621 Georgia Avenue, Silver Spring, Maryland.....	25
Space Technology Laboratory P.O. Box 95001, Los Angeles 45, California Attn: Mr. H. A. Taylor.....	1
VIA: Navy Liaison Office (SP) Air Force Unit Post Office Los Angeles 45, California Attn: M. H. Holt.....	1

CATALOGING INFORMATION FOR LIBRARY USE

BIBLIOGRAPHIC INFORMATION

SOURCE	NOL technical report	CODES		SECURITY CLASSIFICATION AND CODE COUNT	DESCRIPTORS	CODES
		CODES	DESCRIPTIONS			
REPORT NUMBER	62-1149	NOLTR		CIRCULATION LIMITATION	Unclassified	U#31
REPORT DATE	13 August 1962	620119	Ø862	CIRCULATION LIMITATION OR BIBLIOGRAPHIC BIBLIOGRAPHIC (SUPPL., VOL., ETC.)		

SUBJECT ANALYSIS OF REPORT

DESCRIPTORS	CODES	DESCRIPTORS		CODES	DESCRIPTORS	CODES
		CODES	DESCRIPTIONS			
Metals (description)	METAD	Nickel		NICK	Compressible	COMR
Thermodynamic	THED	Lead		LEAD	Equation-of-state	EQUIS
Mirror	MIRR	Tin		TINE	Anomalous	ANOM
Image	IMAG	Thorium		THOI	Behavior	PERF
Metals	META	Titanium		TITN	Comparison	CMBI
Silver	SILV	Thallium		THAL	Mie	MIEQ
Gold	GOLD	Vanadium		VANA	Gruneisen	GRUS
Cadmium	CADM	Tungsten		TUNG		
Cobalt	COBA	Zinc		ZINC		
Chromium	CHRM	Shock		SHOC		
Copper	COPP	Hugoniot		HUGO		
Molybdenum	MOLY	Curve		CURV		

Naval Ordnance Laboratory White Oak, Md.
 (NOL technical report 62-149)
 A COMPLETE E,P,V,T,S THERMODYNAMIC DESCRIPTON OF METALS BASED ON THE P,u MIRROR-IMAGE CONCEPT, by Julius W. Enig. 13 Aug. 1962. 27 p. tables, graphs. Task FR-59. UNCLASSIFIED

Experimental relationship $U=c_0 + a_{H^+}$ and $\alpha \equiv V^{-1}(\partial V/\partial T)P \approx 0$ = constant gives thermodynamic description of metals in which metal appears less compressible than if described by Mie-Gruneisen equation of state. Anomalous behavior of c_p in low pressure neighborhood of initial state excludes simultaneous existance of Hugoniot satisfying U vs. u_H . Relationship and constant in this region. Thermodynamic functions for 16 metals are calculated up to 2 megabars and compared with results obtained from Mie-Gruneisen equation.

Naval Ordnance Laboratory White Oak, Md.
 (NOL technical report 62-149)
 A COMPLETE E,P,V,T,S THERMODYNAMIC DESCRIPTON OF METALS BASED ON THE P,u MIRROR-IMAGE CONCEPT, by Julius W. Enig. 13 Aug. 1962. 27 p. tables, graphs. Task FR-59. UNCLASSIFIED

Experimental relationship $U=c_0 + a_{H^+}$ and $\alpha \equiv V^{-1}(\partial V/\partial T)P \approx 0$ = constant gives thermodynamic description of metals in which metal appears less compressible than if described by Mie-Gruneisen equation of state. Anomalous behavior of c_p in low pressure neighborhood of initial state excludes simultaneous existance of Hugoniot satisfying U vs. u_H . Relationship and constant in this region. Thermodynamic functions for 16 metals are calculated up to 2 megabars and compared with results obtained from Mie-Gruneisen equation.

Naval Ordnance Laboratory White Oak, Md.
 (NOL technical report 62-149)
 A COMPLETE E,P,V,T,S THERMODYNAMIC DESCRIPTON OF METALS BASED ON THE P,u MIRROR-IMAGE CONCEPT, by Julius W. Enig. 13 Aug. 1962. 27 p. tables, graphs. Task FR-59. UNCLASSIFIED

Experimental relationship $U=c_0 + a_{H^+}$ and $\alpha \equiv V^{-1}(\partial V/\partial T)P \approx 0$ = constant gives thermodynamic description of metals in which metal appears less compressible than if described by Mie-Gruneisen equation of state. Anomalous behavior of c_p in low pressure neighborhood of initial state excludes simultaneous existance of Hugoniot satisfying U vs. u_H . Relationship and constant in this region. Thermodynamic functions for 16 metals are calculated up to 2 megabars and compared with results obtained from Mie-Gruneisen equation.

Naval Ordnance Laboratory White Oak, Md.
 (NOL technical report 62-149)
 A COMPLETE E,P,V,T,S THERMODYNAMIC DESCRIPTON OF METALS BASED ON THE P,u MIRROR-IMAGE CONCEPT, by Julius W. Enig. 13 Aug. 1962. 27 p. tables, graphs. Task FR-59. UNCLASSIFIED

Experimental relationship $U=c_0 + a_{H^+}$ and $\alpha \equiv V^{-1}(\partial V/\partial T)P \approx 0$ = constant gives thermodynamic description of metals in which metal appears less compressible than if described by Mie-Gruneisen equation of state. Anomalous behavior of c_p in low pressure neighborhood of initial state excludes simultaneous existance of Hugoniot satisfying U vs. u_H . Relationship and constant in this region. Thermodynamic functions for 16 metals are calculated up to 2 megabars and compared with results obtained from Mie-Gruneisen equation.